Examples of Simplifying Negative Exponents and Fractions

Often in a course such as Math 1P01 or 1P05, when we need to perform a task (e.g., find the critical points of a function), we may need to simplify our expression to do so. This page contains examples of how to simplify fractions and negative exponents, which you as a student may encounter on either the Practice Test or Skills Test.

Example 1:

Simplify the following expression: $\frac{(x-y)^{-1}}{x^{-1}-y^{-1}}$

Important Note: $(x-y)^{-1} \neq x^{-1}-y^{-1}$ so unfortunately this does not simplify to 1.

To simplify expressions involving negative exponents, we often use the fact that $x^{-b} = \frac{1}{x^b}$

So $\frac{(x-y)^{-1}}{x^{-1}-y^{-1}}$

$$= \frac{\frac{1}{x-y}}{\frac{1}{x}-\frac{1}{y}}$$

If we have fractions added or subtracted together like on the denominator, we can find a common base and see if that helps us.

$$= \frac{\frac{1}{x-y}}{\frac{1}{x}\left(\frac{y}{y}\right)-\frac{1}{y}\left(\frac{x}{x}\right)} = \frac{\frac{1}{x-y}}{\frac{y}{xy}-\frac{x}{xy}}$$

$$= \frac{\frac{1}{x-y}}{\frac{y-x}{xy}}$$

We can then invert and multiply to simplify the double fraction.

$$= \frac{1}{x-y}\left(\frac{xy}{y-x}\right) = \frac{1}{x-y}\left(\frac{-xy}{x-y}\right) = -\frac{xy}{(x-y)^2}$$

Example 2:

Simplify: $2+\frac{2}{2+\frac{2}{2+3x}}$

Here we can use a similar technique in simplifying by again bringing the addition or subtraction under a common denominator.

$$=2+\frac{2}{\frac{2(2+3x)}{2+3x}+\frac{2}{2+3x}}$$

$$=2+\frac{2}{\frac{4+6x+2}{2+3x}}=2+\frac{2}{\frac{6+6x}{2+3x}}=2+\frac{2}{\frac{2(3+3x)}{2+3x}}$$

Inverting and multiplying we get

$$=2+\frac{2}{2}\left(\frac{2+3x}{3+3x}\right)=2+\frac{2+3x}{3+3x}$$

We can now find a common denominator and further simplify.

$$=\frac{2(3+3x)}{3+3x}+\frac{2+3x}{3+3x}=\frac{6+6x+2+3x}{3+3x}=\frac{8+9x}{3+3x}$$