# Examples of Simplifying Radicals and Fractional Exponents

We often must simplify expressions to do certain tasks in Math 1P01 or 1P05 (e.g., find intervals of increasing and decreasing). Sometimes this involves simplifying expressions with roots or fractional exponents.

We are often using the laws of exponents in these cases. Some key ones to remember are:

$\sqrt[n]{x} = x^{\frac{1}{n}}$ e.g., $\sqrt{2} = 2^{\frac{1}{2}}$

$x^{-b} = \frac{1}{x^b}$ e.g., $(2-x)^{-3} = \frac{1}{(2-x)^3}$

$x^a x^b = x^{a+b}$ e.g., $(x+1)^2(x+1)^3 = (x+1)^5$

$\frac{x^a}{x^b} = x^{a-b}$ e.g., $(\frac{(x^2-1)^3}{(x^2-1)^2} = (x^2-1)^1 = x^2-1$

$(x^a)^b = x^{ab}$ e.g., $(e^x)^2 = e^{2x}$

Remember we can use any law of exponent "forwards" or "backwards" depending on the question.

## Example 1:

Say we have the expression $\frac{(1+x^3)^{\frac{1}{3}}-(1+x^3)^{-\frac{2}{3}}}{(1+x^3)^{\frac{1}{3}}}$ and wish to simplify it. We assume for this question that $x \neq -1$.

One key thing to identify here is that every exponential expression has $(1+x^3)$ as a base, meaning that it is a common factor. And remember that we can multiply any expression by 1 and it remain the same expression. Mathematics lets us be clever in defining that one. For instance, $\frac{1+x^3}{1+x^3} = 1$ for $x \neq -1$. Multiplying by that expression, however, doesn't really help (Try it and see!).

But we have exponents to the $\frac{1}{3}$ and to the $-\frac{2}{3}$, so that gives us a hint what to try. Notice if we multiply the numerator and denominator by $(1+x^3)^{\frac{2}{3}}$, we get some nice cancellation and simplification!

$$\left(\frac{(1+x^3)^{\frac{1}{3}}-(1+x^3)^{-\frac{2}{3}}}{(1+x^3)^{\frac{1}{3}}}\right)\left(\frac{(1+x^3)^{\frac{2}{3}}}{(1+x^3)^{\frac{2}{3}}}\right) = \frac{(1+x^3)^{\frac{1}{3}+\frac{2}{3}}-(1+x^3)^{-\frac{2}{3}+\frac{2}{3}}}{(1+x^3)^{\frac{1}{3}+\frac{2}{3}}} = \frac{(1+x^3)^1-(1+x^3)^0}{(1+x^3)^1}$$

Recall that $a^0 = 1$ for $a \neq 0$. So $(1+x^3)^0 = 1$ for $x \neq -1$ (which we assumed to be true with this question). So, we can write this as

$$= \frac{1+x^3-1}{1+x^3} = \frac{x^3}{1+x^3}$$

## Example 2:

Say we are asked to rationalize this expression: $\frac{2x}{1+\sqrt{x-2}}$ for $x > 2$ If you recall, rationalizing an expression with a radical means to ensure that the denominator does not have any roots. Again, we will use our tactic of multiplying by 1, where we are creative in defining that one. While $\sqrt{x-2} = (x-2)^{\frac{1}{2}}$, multiplying the numerator and denominator with $(x+3)^{\frac{1}{2}}$ will get us

$$\left(\frac{2x}{1+\sqrt{x-2}}\right)\left(\frac{\sqrt{x-2}}{\sqrt{x-2}}\right) = \frac{2x\sqrt{x-2}}{\sqrt{x-2}+(x-2)^{\frac{1}{2}+\frac{1}{2}}} = \frac{2x\sqrt{x-2}}{\sqrt{x-2}+x-2}$$

This still has a radical on the denominator, so this tactic did not work. We can try something else though: Remember that the difference of squares is $a^2 - b^2 = (a+b)(a-b)$. Can we use this to help us simplify the expression?

What if we multiply the numerator and denominator by $1-(x+3)^{\frac{1}{2}}$? That would give us on the denominator $\left(1+(x-2)^{\frac{1}{2}}\right)\left(1-(x-2)^{\frac{1}{2}}\right) = (1^2 - \left(\sqrt{x-2}\right)^2 = (1-(x-2) = 3-x$

So, $\frac{2x}{1+\sqrt{x-2}} = \left(\frac{2x}{1+\sqrt{x-2}}\right)\left(\frac{1-\sqrt{x-2}}{1-\sqrt{x-2}}\right) = \frac{2x\left(1-\sqrt{x-2}\right)}{1^2-\left(\sqrt{x-2}\right)^2} = \frac{2x-2x\sqrt{x-2}}{3-x}$

We thus end up with an expression with no root in the denominator.