Examples of Solving Exponential and Logarithmic Equations

This document presents a few examples of solving exponential and logarithmic equations.

Example 1:

Solve $2^{3x-4} = 4^{2x+1}$

For questions like these, we are usually using the laws of exponents to get to the solution. We can use these rules either "forwards" or "backwards" depending on the question.

In this example, we should recall two things:

1. $4 = 2^2$ and
2. $\left(a^b\right)^c = a^{bc}$

So, we can write $2^{3x-4} = 4^{2x+1}$ as $2^{3x-4} = (2^2)^{2x+1}$

And then apply the exponent law quoted in 2. to get

as $2^{3x-4} = 2^{2(2x+1)} \Leftrightarrow 2^{3x-4} = 2^{4x+2}$

Now since the base of both exponential expressions are the same, we can take the $\log_2$ of both sides to get $3x - 4 = 4x + 2$

Solving for x we get $-4 - 2 = 4x - 3x$ or $x = -6$

Example 2:

Solve $\frac{6}{2-e^x} = 9$

We can solve this by combining some basic algebra and using the natural logarithm ln.

$\Leftrightarrow 6 = 9(2 - e^x) \Leftrightarrow 6 = 18 - 9e^x \Leftrightarrow -12 = -9e^x \Leftrightarrow \frac{12}{9} = e^x \Leftrightarrow \frac{4}{3} = e^x$

We can then use the natural logarithm ln, which is the inverse function of e^x to get

$$\ln\frac{4}{3} = \ln e^x \Leftrightarrow x = \ln\frac{4}{3}$$

Example 3:

Solve $\ln(x+1) + \ln(x-4) = \ln(x-2)$

Note: This is the natural logarithm or $\log_e x$

For equations with logarithms, we are almost always using (at least in part) the laws of logarithms

1. $\ln(a \times b) = \ln a + \ln b$
2. $\ln \frac{a}{b} = \ln a - \ln b$
3. $\ln a^b = b \ln a$

Like in example #1, we can use these formulas “forward” and “backwards” depending on the question asked. In this example, we can rewrite the equation as

$\ln((x+1)(x-4)) = \ln(x-2)$. We can then raise both sides to the power e.

$e^{\ln((x+1)(x-4))} = e^{\ln(x-2)}$ Since e and ln are inverse functions, we obtain

$(x+1)(x-4) = x-2 \Leftrightarrow x^2 - 3x - 4 = x - 2 \Leftrightarrow x^2 - 4x - 2 = 0$

Now this quadratic does not factor easily but using the quadratic formula we obtain $x = 2 + \sqrt{6}$ and $x = 2 - \sqrt{6}$

One thing to note is that $2 - \sqrt{6} < 0$ and the domain of $\ln\ \mathrm{x}$ is $(0, \infty)$ so the only valid solution to the equation is $x = 2 + \sqrt{6}$.

Whenever you algebraically manipulate to solve an equation, you should always check the solutions found by subbing them back into the original equation to make sure they are valid. Otherwise, you may include incorrect answers.