

Evolving technologies integrated into undergraduate mathematics education

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Focus **Theme 4** and issues raised in **Approaches to Themes 3, 4**, and indirectly **2**.

Abstract

*This submission focuses on the **design of learning environments and curricula** and describes a twenty five year evolution of integrating digital technology³ in the teaching and learning of mathematics at Brock University. It provides information on **actual uses** of technology in university level programs for students, majoring in mathematics, or taking mathematics for their major in another discipline, or aiming to be teachers. A brief history explains the ever increasing use of established mathematics and statistics computer systems in courses and programs until the Department had gained enough experience with technologies to institute a new core mathematics program MICA (Mathematics Integrating Computers and Applications). Student interest in the MICA program is demonstrated by a threefold increase in mathematics majors. The submission pays special attention to the role of the teacher. First, a new faculty member reflects on the teaching adjustments she made to teach in a department that has built an array of technologies into its courses. Second, it explains how technology, in a first year core mathematics course, helps to shift the mediator responsibilities from the teacher to the student. Of particular significance is the students' enthusiasm and willingness to work beyond all expectations on their main project in which they construct Learning Objects.*

Introduction

There are many publications (for examples Kallaher (1), and Baglivo (2)) that describe the integration of established Computer Mathematical Systems (for example Maple (1)) and Computer Data and Statistical Analysis Systems (for example SAS (4)) into mathematics and statistics education at the university level. Because of this wealth of publications and because the Department of Mathematics at Brock University had, by the mid 1990s, integrated such systems in the majority of its courses, we will focus our discussion on the Department's next evolution. We describe how the Department integrated communication technology (e.g. Internet) and environment building technology (e.g. VB.NET (5)), into an innovative core mathematics program called MICA (Mathematics Integrating Computers and

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³ In what follows the term digital will be understood whenever we mention technology.

Applications). For us it is evident that this step was only possible because the majority of faculty had substantial teaching experience in courses that integrate technology in a significant way. The MICA courses provide working examples of mathematics learning environments that integrate technologies. Furthermore, within these courses, students learn how to construct technological environments to explore mathematics. Future teachers have an important place in these courses as they learn to develop technological learning environments that focus on the didactical development of mathematical concepts.

This submission is made by two practitioners who, in the words of the Discussion document, ‘can make solid practical and scientific contributions to ICMI Study 17’. The reader will find; in Section 1 a brief 25 year history of the integration of technologies in mathematics programs at Brock; in Section 2, a discussion of one role that evolving technologies can play in mathematics education; in Section 3, a summary of important aspects of the MICA courses; in Section 4, a description of some didactical considerations that were introduced in MICA specifically for future mathematics teachers, and; in Section 5, a reflection by Buteau on the challenges and adjustments that were required in her teaching in order for her students to achieve the learning expectations of MICA.

Section 1: A brief history of the evolving integration of technologies

In 1985, at the time of the first ICMI Study, the Department of Mathematics was making innovative use of technology in some of its courses. In large enrolment service courses, some faculty (6) were generating individualized sets of problems for each student and Muller (7) was assessing an experimental Calculus course with over 100 students who worked with Maple in a laboratory setting. In this presentation we reflect on the Department’s subsequent sustained development of the use of evolving technologies in its undergraduate mathematics programs. Although one can point to certain years when major changes were implemented, the reality is that evolution and innovation in university mathematics education is a slow process. One reason for this is that few mathematics doctoral programs require teamwork or provide opportunities for reflection on the teaching and learning of mathematics. Yet these experiences are necessary for faculty in a department to work as a team and for its faculty to critically redesign a mathematics program. There is much evidence that technological innovations that are instituted in a course by a single faculty member rarely survive when the course is taken over by another colleague. Therefore, for changes to permeate beyond a set of courses, a consensus needs to be built with the majority of faculty in a mathematics department. The changes that occurred at Brock required many hours of discussions between colleagues and demanded that they approach the subject with open minds. In retrospect, a major argument for the use of technology and for a complete review of the mathematics programs was generated from faculty experiences in Maple laboratories. There they observed student activity and involvement. In general they found that students in laboratories were much

more engaged than in the traditional tutorials and that they were also asking more significant mathematical questions.

By 1995 a majority of students in all mathematics programs were using technology in a significant way. In general students were working with Computer Mathematical Systems or Statistical Analysis Systems. By this time, faculty who were keeping up with the evolution of technologies, especially in the areas of communication and computer environment building, were convinced that learner experiences in mathematics could be further enriched and that these experiences could be structured so as to lead students towards more independence in learning. Over the next five years an innovative core program in mathematics was developed and approved. The philosophy and aims of this program, MICA are described in the Brock Teaching journal (8). Student interest in the MICA program is demonstrated by an increase in mathematics majors from 52 in the first year of the program, 2001, to 140 in 2005. In the following sections we explore how the faculty worked to meet MICA guiding principles, including 1) encouraging student creativity and intellectual independence, and 2) developing mathematical concepts hand in hand with computers and applications.

Section 2: Evolving roles of technology in mathematics education at Brock

In this discussion we describe the evolution of the use of technologies in Brock's mathematics programs. In order to facilitate our points of view we shall use the following definitions :

- **Digital information** – data, algorithms, responses, etc. that are available through technologies;
- **Knowledge** – the acquaintance of information obtained through experience or instruction;
- **Understanding** – the power or ability to acquire and interpret knowledge.

A principal aim of integrating technologies into mathematics programs at Brock is to teach students how to transform information into understanding. Initially the teaching and learning process matched the one that the Department used before the birth of digital information, namely

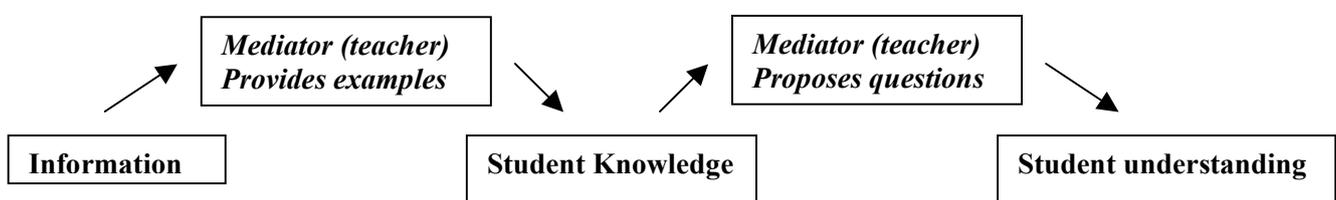


Figure 1

This model also works well with mathematical technologies such as Maple, Mathematica (9), Minitab (10), SAS, etc... These are more than repositories of information, they are intelligent⁴, in the sense they are capable of generating new information. A challenge for undergraduate mathematics education continues to be

⁴ In this text we use the term 'intelligent' to distinguish from technologies that are passive, i.e., strictly provide data.

that such systems can, for the knowledgeable user, provide solutions to most well structured problems that arise in the first three years of a traditional university mathematics program. The integration of these technologies into the Brock mathematics programs changes the first box in *Figure 1* and adds digital information to traditional forms of information (texts, lecture notes, etc.). This addition provides many ways in which to enrich the base of student knowledge, for example: faculty can spend more time on the development of mathematical concepts because they and the students can rely on the technology to provide technical information; alternative representations are often easily generated; students can work on problems and applications that are not bound by traditional course information, and; learners can explore conceptually advanced mathematical concepts which are normally deferred until all the analytical skills have been addressed. In summary, by the mid 1990s, information technologies were well established in a majority of mathematics Brock courses.

By that time some faculty became aware of the great potential of communication environment building technologies. Their vision was that a program would be developed to integrate these and to motivate its students to take on, more and more, the responsibility of mediator in their own mathematics learning (second and fourth box in *Figure 1*). Ralph (11) summarizes the situation as follows: “The central challenge of any mathematics program is to create an environment in which students become internally directed and personally invested in moving themselves along the long road to mastery. The problem with traditional undergraduate mathematics programs in this regard is that if students try to take the initiative in creating and investigating problems and applications of their own devising, they quickly come up against difficulties that they cannot handle with purely analytical tools. For this reason, traditional programs must be very tightly choreographed around the problems that can be solved by hand and over the years this approach can become “canned” and regimented. Technology can offset the rigidity of a traditional mathematics program by providing students with access to an endless supply of problems and applications that can be investigated both computationally and analytically.” Therefore the aim of the proposed MICA program was to change the model in *Figure 1* to the following:

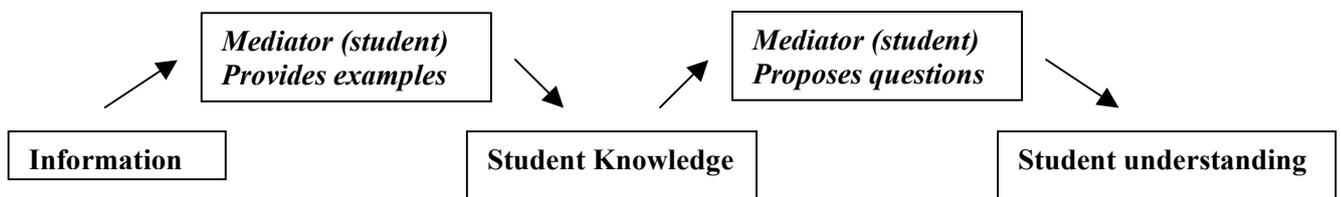


Figure 2

In many ways this parallels the development that one would hope for in a mathematics program that has a core of modelling courses. However students in the MICA program build on a knowledge base that is more extensive as the information they have access to includes both passive and intelligent digital sources.

How does one educate a student to become her own mediator and how is this done as early as possible in their university mathematics experience? At Brock students take, in their first semester, a course in Calculus and one in Linear Algebra. Both of these courses include extensive experiences with Maple and with Journey Through Calculus (12). A brief discussion on the first MICA course that students take in the second semester and upper year MICA courses will highlight the approach that the Department has taken to encourage creativity and intellectual independence as the students develop mediator skills.

Section 3: MICA courses – directions and the integration of technologies

In the first part of the MICA I course, students are exposed to a rich context for conjectures: prime numbers and Collatz conjecture. During lectures⁵, students work in small groups of 3 or 4 and raise original questions and conjectures about the topics. These are written on the board and a discussion on their testability follows. For their first assignment, each student designs a program (vb.NET) in which they explore a conjecture of their own. In the second part of the course, students are introduced to modular arithmetic leading to the theory behind RSA encryption. The speed of the theoretical presentation is determined by the students as they lead the way by making observations and conjectures from explorations, computations and theorems. Of course the lecturer guides students but importantly he/she reacts to class questions/ observations/ conjectures that are constantly encouraged and raised. Students then implement the complete algorithm of RSA encryption. The last topic in the course is discrete and continuous dynamical systems. Each student designs a program that outputs numerical values and graphs the cobweb diagrams of the logistic function. This topic concludes with an exploration, in the lab, of the system stability which students simultaneously test and visualize the theory with their own program.

A major part of MICA I is the original final project that encompasses a computer program and a written report. Each student selects a mathematical topic in which they are particularly interested and intrigued. Mathematics students focus on a mathematical investigation. Future teachers design a learning program about an elementary or high school mathematical concept. Students from another core discipline investigate a mathematical application to their own discipline. In this project students essentially construct and implement a Learning Object — an instructional component that focuses on one or two mathematical concepts and that is designed for another person. These objects are interactive, engaging, easy to use, and are designed to mediate the user from information to understanding. In the MICA program Learning Objects may include exploration of a mathematical conjecture or of a mathematical application. The main goal in MICA I course is to bring students to experience becoming the mediator through the design of original Learning Objects.

⁵ Presently the course runs with two hours of lectures and two hours of labs per week. The experience of the Department is that this type of course works best with a maximum enrolment of 35 students in each section.

The first experimental project on Learning Objects at Brock was undertaken in the summer of 2002. It involved a team composed of mathematics professors, practicing mathematics teachers, future teachers, and mathematics and computer science students. Examples of finished products can be viewed on the departmental website (13). Other examples of Learning Objects developed by students in the MICA courses are also available (14).

It is our experience that the construction of a Learning Object not only builds on the designer's mathematical and didactical knowledge but it reveals these understandings in a visual and interactive way.

In the MICA II full year course the focus is on mathematical modelling of diverse types including, for example, discrete dynamical systems, stochastic models, Markov chains, empirical models, and queuing models. These topics, covered in the *MICA way*, are all implemented (mainly in VB.NET and Maple) by students and are concluded with simulation and conjectures. For example, students design (VB.NET) a Learning Object to explore the distribution histograms and graphs of random variables. This is done before the students see the Central Limit Theorem in their Statistics course. Therefore students are guided through different computations, and are asked to develop conjectures based on their observations. Not all students are able to conjecture the theorem on their own, but after a full class discussion about plausible conjectures they are able to identify examples of the theorem in their results. When students finally see the theorem in their statistics class, it is no longer a theorem *outside* them, but indeed, it is somehow *internalised* since they personalized it within the design and use of their Learning Object. MICA II students work on two main original projects for which they personally decide on a topic. Their projects are significantly more sophisticated than in MICA I, since they have a better mastery of the technology and importantly, they have become more confident in their role of mediator.

The MICA III full year course is focussed on partial differential equation modelling including for example heat flow and wave propagation. Guided assignments and projects (mainly in Maple and C++(15)) each include an original part in which students have to fully use their role of mediator. For example, students were assigned to extend and improve some MAPLETs (3) that animate solutions of particular PDEs. Two students presented their remarkable MAPLET extension at the Maple Summer Workshop (3) in Summer 2004. With mastery of technology and with their ability to mediate their own learning, undergraduate students can contribute to the development of new mathematics.

Section 4: Technologies and the education of future mathematics teachers

Teacher Education in Ontario follows a consecutive model. This means that individuals interested in teaching must first graduate with a university degree and then apply to a Faculty of Education for a one-year program. After completing this additional year they receive a teaching certificate. For future elementary and middle school teachers and for future mathematics teachers at the secondary level, the

consecutive model clearly places important responsibilities on departments of mathematics. How can these future teachers be best educated in mathematics to meet their specific and desired goals? Unfortunately many universities do little more than to pay lip service to this population of students. Within the Brock community, Muller has been proactive in negotiations across different Faculties in order to develop and establish concurrent education programs. In these programs, students follow integrated studies between a Faculty that offers a teachable subject⁶ and the Faculty of Education. For those students who enter university with a desire to become teachers, concurrent programs provide opportunities to reflect on didactical issues starting from their first undergraduate year.

The Mathematics Department at Brock has taken its responsibilities for future teacher education very seriously and has developed programs or courses for all levels of school teaching. Appropriate technologies such as Geometer's SketchPad (16) and other Ministry of Education school licensed programs are used in appropriate courses. Concurrent education students who aim to specialize in mathematics and to be certified for teaching at the middle and high school levels, take a majority of the MICA courses which play a fundamental role for them. They provide a unique opportunity for these future teachers to reflect on their own development as a mediator. Furthermore in their MICA projects they construct Learning Objects which have strong didactical components.

Section 5: Reflections by a new faculty, Chantal Buteau

I am currently in my second year as faculty in the Department and I'm coming from a rather traditional mathematics education⁷. I knew that I was joining a department that makes extensive use of technologies in its courses. Therefore I had mixed feelings, anxiety, insecurity and excitement. In service courses (Calculus and Statistics for large classes) my main concern was and is to focus on concepts rather than on computational techniques that can easily be handled using technology. I admit that it is a constant battle for me. When I was taught these concepts there was equal emphasis on concepts and computation abilities. Diverse and rich discussions with colleagues help me to find a good balance. Also, my class preparations keep changing as I rethink what should be first discovered by students in a guided assignment using technology rather than directly presented to them. My conception of assignments and exams also had to be changed. As a new lecturer, it has been a genuine and enriching challenge not to *copy* the teaching model I had experienced.

In the MICA I course I faced teaching an innovative course in which the *how* to present the theory was more important than the *what*. On top of this, the *how* was

⁶ Teachable subjects are specified by the Ministry of Education as being appropriate major disciplines for future teachers

⁷ Undergraduate and graduate courses, towards my PhD in mathematics, were relatively traditional and rarely required the use of technology. My mathematics teaching experience was mainly as a teaching assistant, also in a traditional setting (no use of computer software). However my main research discipline (mathematical music theory) has required me to make extensive use of computer programming (JAVA (17) and Mathematica).

supported by a programming environment. Fortunately, during my PhD, I had experienced some experimental mathematical investigations supported by technology. This was my beacon together with uncountably many discussions with my colleague Bill Ralph who has been teaching this course since the MICA program was first launched. It did take me some time to understand my role in the course. How can I best assist the students to become the mediator of their own mathematical development? What mediation should or should not be provided at any particular time? I had to adjust to the fact that a class can sometimes take a direction different from what I had planned. This is not a secure position for a fairly new lecturer, as I had to build on class interactivity and not reject it. I challenge students to explore mathematics on a personal level. Students challenge my traditional education of mathematics teaching.

The astonishing pride of MICA I students for their final projects confirmed that the department is for me a great environment for learning how to teach mathematics in the XXI century.

Conclusions

Technologies are evolving so rapidly that there are many avenues that mathematics departments can take to integrate them into their mathematics programs. This submission describes one route that the Department at Brock has taken to structure technological environments to help students engage in abstract mathematics. We have found that the approaches, activities, and experiences in the MICA courses are able to harness the students' motivations thereby empowering them to become their own mediators in the development of their mathematical knowledge and understanding.

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