

Motivic Spaces of Scores through RUBATO's MeloTopRUBETTE

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Abstract

In the framework of the Distributed RUBATO platform, we propose an improved version MeloTopRUBETTE of RUBATO's successful module MeloRUBETTE for motivic analysis. It fits with computational steering interaction and implements topological and sheaf-theoretical aspects of motive theory, such as motivic evolution trees (MET) and stalk dimensions for weight function sheaves. We present the theory and the algorithm flow chart of our topological model of motivic analysis following Rudolph Reti's approach.

Key Words: Motive Analysis, Motivic Topologies, Mathematical Music Theory, Melo(Top)RUBETTE module, RUBATO software, Quantification of Topologies, Function Spaces, Weight Functions.

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1 Introduction

We propose an improved version of the module MeloRUBETTE of the software platform RUBATO for musical analysis and performance [9], [15]. This module of the now developed Distributed RUBATO platform [11] (chapter 40) is called MeloTopRUBETTE¹ and implements our mathematical motive theory. Its design follows the requirements of computational steering, i.e. with interactive control of the computational process during its execution

The motivic analysis of a score has always been a main subject in musicology, but the very complexity of the motivic concepts and configurations made it impossible to 'traditional' musicologists to transcend pure intuition. Nonetheless, crucial ideas about motivic analysis were proposed by Rudolph Reti and Michael Kopfermann [13]. In order to find out which motives are the germs and motors in the evolution of the motivic content, they suggested an immanent approach: One should not impose the germinal motives from outside, but construct the germs from a thorough analysis of all possible motif structures and relations within a given composition. So musicology is facing two main problems: To formalize the concept framework dealing with motives, and to establish a formally valid model of motivic germs.

We propose a solution to these two problems: In [5], we introduced a model which formally conceptualizes Reti's and Kopfermann's approach. The present operationalization of our motivic theory implements modular algorithms, making possible to modify the model during its empirical testing process.

Let us briefly recall our mathematical model on motivic analysis of music. In [8], a mathematical theory of musical structures was developed which is based on local charts and global discrete varieties of tones in specific parameter spaces. In this approach, a motif is a local chart of a specific type. A motivic interpretation of a musical composition is a global variety, i.e. an interpretation of the score by a selected covering system of motives. Topological spaces built on motivic interpretations are called motivic spaces. They are defined from orbits of motives under actions of transformation groups, from metrical similarity between motives with same cardinality, and from submotif relations. It is important to observe that this topological structure is defined on the set of all motives (with different cardinalities) of a score. The motivic space of a score is however non-intuitive (not Hausdorff, only

¹The name originates from '**RUBETTE** of **Melodic Topologies**'.

of type T_0). For this reason, a more geometric perspective has been realized in the original MeloRUBETTE by giving each motive, and thereby each tone of the score, a weight that corresponds to their topological "presence" and "content", see chapter 22.9 in [11]. Corresponding analytical results have been used to investigate and produce performances of classical compositions [1], [2], [3], [4], [10], [14].

The problem of geometrization of motivic spaces deals with the main problem of motive theory: the exhibition of germinal motives. It has been further investigated and led to the concept of a motivic evolution tree (MET), the graphical representation of an overall spectrum of a score's motivic structure (motivic space) [6]. The first trace of a sheaf-theoretic perspective related to the MET is given in [7]. In fact, the sheaves give rise to coordinate functions (the global sections) which yield embeddings of motivic topologies in real vector spaces. First empirical investigations [6], [7] on small compositional units, such as the main theme of Bach's "Kunst der Fuge", demonstrated the viability of our approach. The present MeloTopRUBETTE opens the path to deeper and more meaningful empirical investigations.

As a major improvement of the MeloTopRUBETTE against the MeloRUBETTE is the interactive control of the ongoing computational process making possible to extend and to improve the model "on the flight". We also mention that the contour similarity "theory" of the American Set Theory is a special case of our implementation, in which their contour similarity concepts are extended to a topology on the space of all motives of a score, i.e. a structure in which a similarity concept between motives of different cardinalities is introduced. See section 5 for a more detailed comparison between these two RUBETTES.

2 Motivic Spaces

We shortly summarize the construction of a motivic space of a score which is here simplified, and for which details can be read in [5], [7],[11].

The formal definition of a motivic space presupposes a motif concept. We want to restrict our attention to a minimal parameter setup in order to make the essential clear, and we refer the reader to the end of this section for a simple example leading to a motivic topology. Consider the space $\mathbb{R}^{\{O,P,D,L,G,C\}} \cong \mathbb{R}^6$ of tone parametrization for which the parameters are respectively *onset*, *pitch*, *duration*, *loudness*, *glissando* and *crescendo*, and

consider also the canonical projection

$$p_O : \mathbb{R}^{\{O,P,D,L,G,C\}} \rightarrow \mathbb{R}^{\{O\}}$$

on the axis of onset events. Denote $\mathbb{R}^{OP\dots} \subseteq \mathbb{R}^{\{O,P,D,L,G,C\}}$ the space of notes parametrized by at least onset and pitch parameters. A *motif* $M = \{m_1, \dots, m_n\}$ is a non-empty finite subset in $\mathbb{R}^{OP\dots}$ such that $p_O(M)$ is a bijection.

A motif is therefore a finite set of notes $m_i = (o_i, p_i, \dots)$ such that only one note is heard at a given onset. A submotif M' of a motif M is a motif such that $M' \subset M$. The set of all motives is denoted by MOT , which is the disjoint union of subsets MOT_n of all motives M of cardinality $card(M) = n$. If we are given a score S , a collection of motives with all notes living in S is denoted by $MOT(S)$, which is the disjoint union of the subsets

$$MOT_n(S) = MOT(S) \cap MOT_n.$$

Motives are always mapped to abstractions, for example for contour information. This means that we have a family $t = (t_n)$ of maps

$$t_n : MOT_n \rightarrow \Gamma_{t,n}$$

into mutually disjoint² sets $\Gamma_{t,n}$ of *abstract motives of abstract cardinality n*. The family t is called the *shape type*, whereas the elements of $\Gamma_{t,n}$ are called *abstract motives of abstract cardinality n*. A typical such map is the contour type $t = Cont$ (which corresponds in the American Set Theory to the *COM matrix*). We have

$$\Gamma_{Cont,n} = \mathbb{Z}^{n(n-1)/2},$$

and if $M = \{m_1, m_2, \dots, m_n\} \in MOT_n$ with notes $m_i = (o_i, p_i, \dots)$, we set $t_n(M) = (\Delta_{ij})_{1 \leq i < j \leq n}$, with $\Delta_{ij} = 1$ if the pitch difference $(p_j - p_i)$ of notes m_j and m_i in M is positiv, 0 if the difference is null, and -1 if it is negativ; see [7] for further examples. On each space $\Gamma_{t,n}$ of abstract motives of abstract cardinality n , we suppose also given a pseudo-metric $d_n(x_1, x_2)$, for example the Euclidean metric on $\Gamma_{Cont,n}$. Call the family $d = (d_n)_{n \in \mathbb{N}}$ a pseudo-metric on the shape type t . This induces a pseudo-metric (family) $d_t = (d_{t,n})_{n \in \mathbb{N}}$, which on each MOT_n is defined by $d_{t,n}(M_1, M_2) = d_n(t(M_1), t(M_2))$.

Finally, we suppose that there is a pair of t_n -equivariant group actions $P \times MOT_n \rightarrow MOT_n$, $P \times \Gamma_{t,n} \rightarrow \Gamma_{t,n}$ for each n . Following Nattiez and Ruwet

²In the general theory, disjointness is not mandatory, however

[12], the group P is called the *paradigmatic group*. The typical example is the pointwise action of the affine counterpoint group CP , which (1) on the space $\mathbb{R}^{OP\dots}$ of notes acts as the group of affine transformations generated by all the translations, the horizontal reflexions U_p at pitch p (the inversions), and the vertical reflexions K_o at onset o (the retrogrades); and (2), on $\mathbb{Z}^{n(n-1)/2}$ is the canonically induced action (translations act trivially, inversions by sign inversion, and retrogrades by sign inversion and index exchange). One also supposes that P acts as a group of isometries³ (preserving metrical distances) on each $\Gamma_{t,n}$, which clearly induces an action by isometries on MOT_n . We call the inverse image $t^{-1}(P \cdot t_n(M))$ of a P -orbit of a motive's abstract motif $t_n(M)$ its *gestalt*:

$$Ges(M) = t^{-1}(P \cdot t_n(M)).$$

The gestalts define a partition of the total space MOT , the set of gestalts is denoted by GES . It is evidently the disjoint union of the classes GES_n in MOT_n . Its trace in $MOT(S)$ is denoted by $GES(S)$, and the projections are denoted by $\gamma : MOT \rightarrow GES$, and $\gamma(S) : MOT(S) \rightarrow GES(S)$. We have a (family of) pseudo-metric(s) $Gd_t^P = (Gd_{t,n}^P)_{n \in \mathbb{N}}$ with $Gd_{t,n}^P$ on GES_n , which is defined by

$$Gd_{t,n}^P(Ges(M_1), Ges(M_2)) = \inf_{p \in P} d_{t,n}(p \cdot M_1, M_2).$$

We now are ready to set forth the topological framework. For a given data set as described above, suppose that $\varepsilon > 0$ is a real number, and that $M \in MOT_n$. Then we set

$$U_\varepsilon(M) = \{N \mid \exists N' \in MOT_n, N' \subset N : \inf_{p \in P} d_{t,n}(p \cdot M, N') < \varepsilon\},$$

and call it the ε -neighborhood of M . Observe that it is at this point that we link motives of different(!) cardinalities. If our setup fulfills the inheritance property [5], [7], the system of ε -neighborhoods defines a base for a topology on MOT . For example, the contour type together with the Euclidean metric fulfills the inheritance property. The space MOT with this topology is called the *motivic space*, its relativization to $MOT(S)$ is called the *motivic space on S* .

We now introduce a topology on GES . To this end, we need a relation on gestalts corresponding to the *submotif relation* on motives: If G and G'

³Note that this hypothesis is natural: the distance between two motives should be the same as when one e.g. equally translates both of them!

are two gestalts, we say that G' is a *small gestalt* of G , in signs: $G' \sqsubset G$, iff there are motives $M \in G$, $M' \in G'$ such that $M' \subset M$. Then the sets

$$U_\varepsilon(G) = \{H \mid \exists H' \in GES_n, H' \sqsubset H : Gd_{t,n}^P(G, H') < \varepsilon\}$$

form a topological base of GES . With this topology (the structures leading to this topology be implicitly assumed), GES is called a *motivic gestalt space*, for a score S the relative space $GES(S)$ is called a *motivic gestalt space on S* . We denote the ε -neighborhood of a gestalt G in $GES(S)$ by

$$SU_\varepsilon(G) := U_\varepsilon(G) \cap GES(S),$$

and denote $mult(G)$, called the *multiplicity* of G , the cardinality of all motives in $MOT(S)$ in gestalt G .

Theorem [5]. If the pseudo-metric Gd_t^P is a metric (i.e., if this is the case for all n), then the canonical maps $\gamma : MOT \rightarrow GES$, and $\gamma(S) : MOT(S) \rightarrow GES(S)$ are open continuous maps onto the quotients, and the topologies on GES and $GES(S)$ are the quotient topologies.

Example. We exemplify the setup leading to a motivic topology. See [6] for a more complete description. Consider the space $\mathbb{R}^{\{O,P,D\}}$. We fix the parameters O, P , and D in a way which is standard in Mathematical Music Theory [8]: For the pitch values, we select the usual gauge with $C_4 = 0$, and the chromatic pitch set being parametrized by the integers, i.e. $C_{\sharp 4} = D\flat_4 = 1$, $D_4 = 2$, etc. Duration values are taken by the prescription that 1 in the O -coordinate corresponds to the literal mathematical value of 4/4 duration. The first tone of a score is given onset value 0. Consider Figure 1.

We suppose that our score S contains only the eight notes from Bach's *Kunst der Fuge* 8-tone Main Them as shown in Figure 1 top bars. First we have the set of the score's notes:

$$S = \left\{ \begin{array}{l} (0, 2, \frac{1}{2}), (\frac{1}{2}, 9, \frac{1}{2}), (1, 5, \frac{1}{2}), (\frac{3}{2}, 2, \frac{1}{2}), \\ (2, 1, \frac{1}{2}), (\frac{5}{2}, 2, \frac{1}{4}), (\frac{11}{4}, 4, \frac{1}{4}), (3, 5, \frac{1}{2}) \end{array} \right\}$$

We select the collection $MOT(S)$ of motives for the score S as containing all motives with cardinality between 2 and 4. Therefore, the collection $MOT(S)$ contains $\binom{8}{2} = 28$ motives with cardinality 2, $\binom{8}{3} = 56$ with cardinality 3, and $\binom{8}{4} = 70$ with cardinality 4, which makes a total of 154 motives.

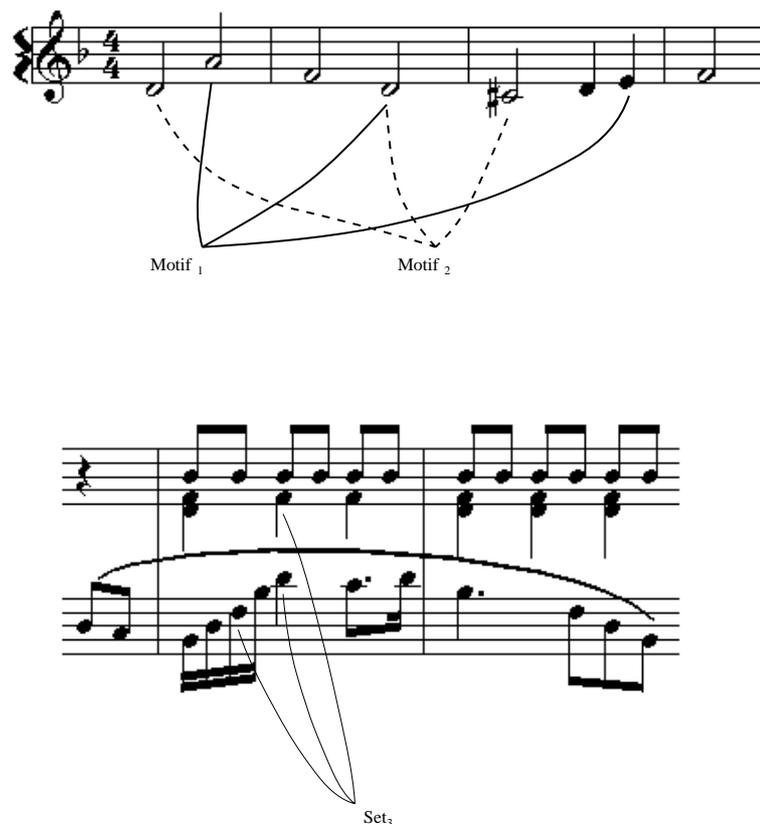


Figure 1. Examples of sets of notes which form a motif: $Motif_1$ and $Motif_2$; and which do not form a motif: Set_3 .

We consider the two motives $Motif_1 = \{(\frac{1}{2}, 9, \frac{1}{2}), (\frac{3}{2}, 2, \frac{1}{2}), (\frac{11}{4}, 4, \frac{1}{4})\}$ and $Motif_2 = \{(0, 2, \frac{1}{2}), (2, 1, \frac{1}{2}), (\frac{3}{2}, 2, \frac{1}{2})\}$ from Figure 1. The abstract images of $Motif_1$ and $Motif_2$ are $Cont_3(Motif_1) = (-1, -1, 1)$ and $Cont_3(Motif_2) = (0, -1, -1)$. The gestalt for the counterpoint paradigmatic group CP of $Motif_1$ is the collection of all motives in $MOT_3^*(S)$ such that their images through the mapping $Cont$ is one of the following four abstract motives: $(-1, -1, 1)$, $(1, 1, -1)$, $(-1, 1, 1)$, or $(1, -1, -1)$ (corresponding respectively to the abstract motif $Cont_3(Motif_1)$, its inversion, its retrograde, and its inversion composed with the retrograde). Using the *MeloTopRUBETTE* for identifying the motives together, we get the following number of gestalts: there are 2 gestalts with motif cardinality 2, 5 with cardinality 3, and 18 with car-

dinality 4.

The Euclidean distance between the two motives $Motif_1$ and $Motif_2$ is

$$d_{Cont}(Motif_1, Motif_2) = ((-1 - 0)^2 + (-1 - -1)^2 + (1 - -1)^2)^{1/2} = \sqrt{5}$$

and the Euclidean distance between their respective gestalts is

$$Gd_{Cont}^{CP}(Ges(Motif_1), Ges(Motif_2)) = \min\{\sqrt{5}, \sqrt{5}, 3, 1\} = 1.$$

Finally, given an $\varepsilon > 0$, the ε -neighborhood of $Motif_1$ is the set of all motives M in $MOT(S)$ with cardinality 3 or 4 such that:

1. If $card(M) = 3$, then

$$\min \left\{ \begin{array}{ll} d_3(Cont(M), (-1, -1, 1)) & , d_3(Cont(M), (1, 1, -1)), \\ d_3(Cont(M), (-1, 1, 1)) & , d_3(Cont(M), (1, -1, -1)) \end{array} \right\} < \varepsilon;$$

2. If $Card(M) = 4$, then there is a submotif $M' \subset M$ with cardinality 3 such that

$$\min \left\{ \begin{array}{ll} d_3(Cont(M'), (-1, -1, 1)) & , d_3(Cont(M'), (1, 1, -1)), \\ d_3(Cont(M'), (-1, 1, 1)) & , d_3(Cont(M'), (1, -1, -1)) \end{array} \right\} < \varepsilon.$$

By definition of ε -neighborhoods, motives with cardinality 2 cannot be in the neighborhood of $Motif_1$.

This completes the setup of the motivic composition space $MOT(S)$ of the score S .

3 Presence, Content, and Weight Functions

Motivic gestalt spaces are [5] of type T_0 , ‘almost’ of type T_1 , and, if $MOT(S)$ contains motives with different cardinalities, not at all of type T_2 (Hausdorff), which excludes any intuitive representation of the topological structure. Therefore, in order to provide us with a more geometric picture of the motivic spaces and motivic gestalt spaces on a score S , we introduce real-valued functions which account for the topological relations on these spaces. Observe that we have to take into account the intrinsic neighborhood asymmetry between gestalts. For more details about quantitative functions, refer to [7].

The 'presence' of a gestalt is the magnitude of its neighborhood and its 'content' is the frequency of its appearance in other motives' neighborhoods: First consider two gestalts G and H in $GES(S)$ with $G \in GES_n$, and a neighborhood radius $\varepsilon > 0$. If $H \in SU_\varepsilon(G)$, then one measures the presence of gestalt G in gestalt H (or, inversed roles: H being contained in G) by the intensity integer

$$Int_\varepsilon(H|G) = card\{H' \prec H \mid H' \in GES_n \cap SU_\varepsilon(G)\} \cdot mult(H)$$

Since the higher cardinality difference between G and H the higher the probability that $Int_\varepsilon(H|G) \neq 0$, we weight the intensity by $1/2^{(card(H)-card(G))}$. The *presence and the content of G at radius $\varepsilon > 0$* is defined ⁴ as

$$Presence_\varepsilon(G) := \sum_{H \in GES(S)} 1/2^{(card(H)-card(G))} \cdot Int_\varepsilon(H|G)$$

$$Content_\varepsilon(G) := \sum_{H \in GES(S)} 1/2^{(card(G)-card(H))} \cdot Int_\varepsilon(G|H)$$

and the *weight of gestalt G at radius ε* is

$$Weight_\varepsilon(G) := Presence_\varepsilon(G) \cdot Content_\varepsilon(G)$$

Note that these *quantification functions*, presence, content and weight functions, on gestalts can be extended to motives, such as $MWeight_\varepsilon : MOT(S) \rightarrow \mathbb{R}$, and from which we then define the *weight of a note n of the score S at radius $\varepsilon > 0$* :

$$NWeight_\varepsilon(n) := \sum_{n \in M \in MOT(S)} MWeight_\varepsilon(M).$$

The information related to S , which is set forth by the quantification functions, is articulated in sheaves of function vector spaces F on $GES(S)$, whose sections $F(U)$ are linear combinations of determined systems of presence functions, content functions, or weight functions, respectively: Consider, for a given $\varepsilon > 0$, a quantification function f_ε . We define a presheaf F' through

$$F'(GES(S)) = \mathbb{R} \langle f_\varepsilon, \varepsilon \rangle$$

⁴There are more parameters in the general definition [7] of these two functions.

and for any $\varepsilon' > 0$ and gestalt G in $GES(S)$

$$F'(U_{\varepsilon'}(G)) = F'(GES(S)|U_{\varepsilon'}(G)).$$

The sheafification F of F' for presence, content, and weight, respectively, constitutes a system of local coordinate functions on motivic gestalt spaces.

4 The Flow Chart for the Calculations in the MeloTopRUBETTE

Our topological model of motivic analysis of music is implemented in the MeloTopRUBETTE according to the flow chart shown in Figure ?? . The blue ellipse of the flow chart is the link from the core program to the whole software RUBATO. It implies a change of programming language since Rubette's core programs are normally written in java whereas the internal language of the RUBATO platform is the denotator language [?]. Each pink box represents some calculations from which meaningful objects for the model are created. These objects are sent back to RUBATO, and that is why the pink boxes are provided with an output denotator for visual and/or sonic representation on the RUBATO's PrimaVista Browser [11] (chapter 40). Blue Ellipses are not present in the flow chart for a simpler visualization of the whole architecture. Green boxes are mainly intermediate calculations for constructing the motivic topologies, and the blue ones are linear algebra operations. The lozenges are tests for stopping their respective loops. The white boxes are initialization of cycle variables.

There is a double input. On one side, input (00) contains a score S as a denoTeX or MIDI file. On the other side, input (01) contains 20 input parameters: the score's collection of motives parameters (e.g. minimal and maximal motif cardinality), the topological parameters (e.g. shape type and paradigmatic group), the similarity radii set parameters (in order to obtain relevant radii for evaluating the quantification functions), and the optional output parameters (e.g. with or without motivic evolution tree).

MeloTopRUBETTE's Core Flow Chart

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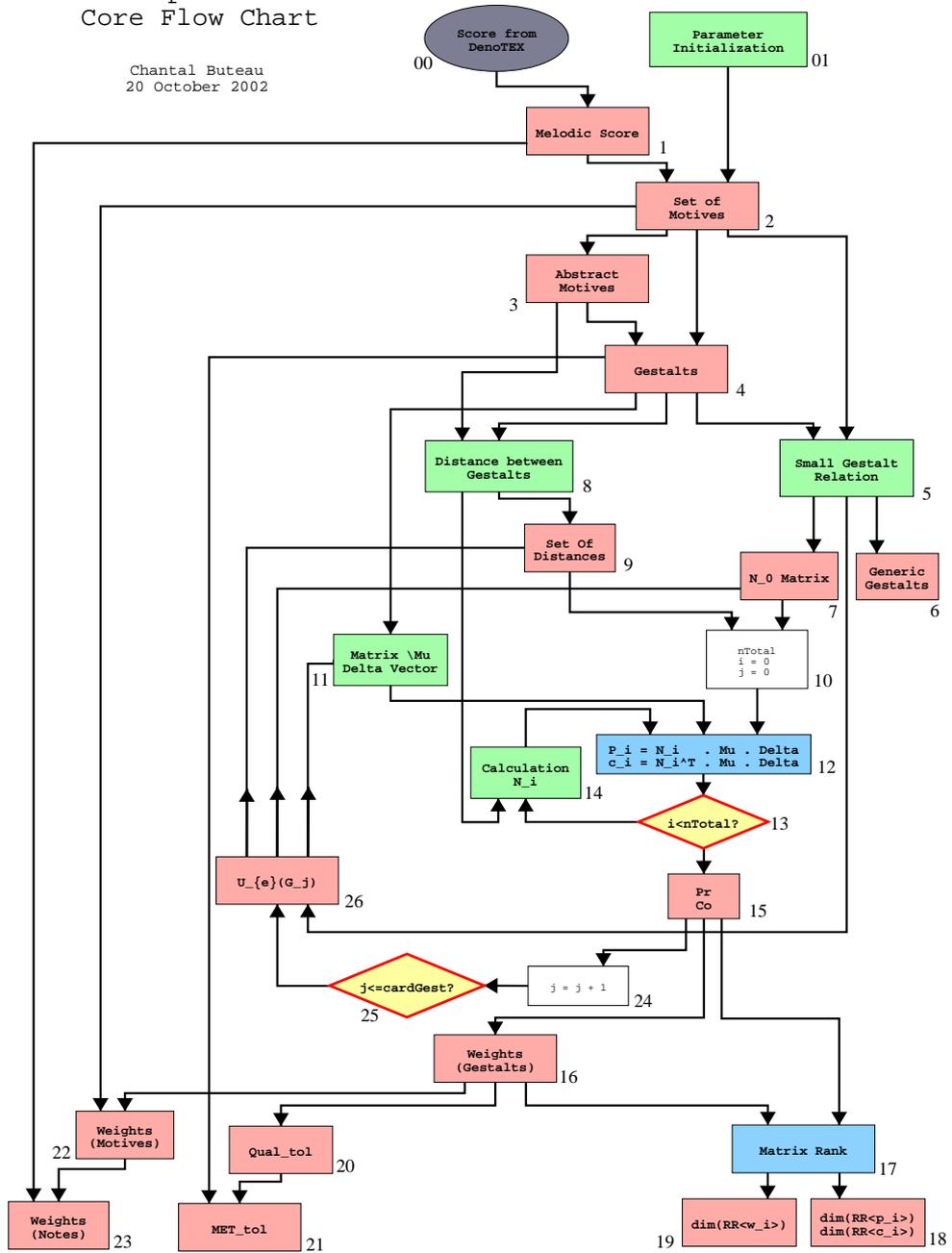


Figure 2. The flow chart of the MeloTopRUBETTE.

In (1) the score S is simplified into a collection of notes parametrized by their onset and pitch values. Observe that any other events such as pauses are not present in this simplified *melodic score*. The collection of all motives of the score to be analysed is created in (2). The abstract images (shapes) of the motives are calculated in (3). The important step (4) corresponds to the identification of motives (from (1)) into gestalts (classes of motives) with respect to an action of a paradigmatic group. In (5) the small gestalt relation between all pairs of gestalts (from (4)) is evaluated. Note that this relation on gestalts is the corresponding relation to the submotif relation on motives, and it is exactly after this step that all other calculations are done on gestalts instead of on motives, until we finally come back, at (22), to the level of motives.

Directly resulting from step (5) the collection of generic gestalts, i.e. the maximal elements of the gestalt composition space $GES(S)$ for the score S under the topological dominance relation, is created in (6), and the N_0 -matrix in (7). The N_0 -matrix of small gestalt relations corresponds to the ε_0 -neighborhoods, $\varepsilon \rightarrow 0$, of all gestalts.

The distance function between all pairs of gestalts with same cardinality is evaluated in (8), as well as the collection of all changing epsilons, i.e., those neighborhood radii where neighborhoods (they are all finite and only differ for specific "jumping" radii) become larger while increasing the radii. Box (9) is the unit where the collection of all neighborhood radii at which quantification functions, i.e. presence, content and weight functions, will be evaluated, is created.

The boxes from (10) to (14), and from (24) to (26) comprise a cycle for the calculation of dimensions of function spaces and of stalk dimension of function spaces for content and presence. More precisely, in (10) the presence/content loops variables are initialized. The running variable i deals with the enumeration index of neighborhood radii from (9), and j with the enumeration index of gestalts in S . In (11) the gestalt multiplicities as well as the linear combination gestalt coefficients are initialized as respectively the μ -Matrix and Δ -vector. Steps (12)-(15) correspond to the presence/content loop first ($j = 0$) on the whole motivic composition space, and then restricted on the ε_0 -neighborhood of the j th Gestalt. In fact, linear operations in (12) evaluates the presence and the content of each gestalt at a given neighborhood radius. It underlines the reciprocity property of these two quantification functions as we can see by comparing their respective linear operations. Step (13) checks if all neighborhood radii from (9) has been evaluated in (12),

and in (14), the N -matrix is modified with respect to the growth of the gestalt neighborhoods for a larger radius. The resulting evaluated presence and content functions at all radii, in the form of a matrix, is finally given in (15).

Weights of gestalts can then be calculated in (16), again in the form of a matrix. In (17)-(19) the dimension of each quantification function vector space is calculated by a linear algebra operation.

The qualitative function is evaluated in (20) which then leads to the motivic evolution tree (MET) in (21). We recall that the MET of a score S is a graphical representation of the overall motivic spectrum of S with respect to the chosen parameters for the analysis.

At (22) we go back to the motif level by assigning to each motif its respective weight. Weights of notes are calculated in (23).

Finally steps (24)-(26) (together with (7),(9)-(15)) correspond to the presence/content sheaf loop. More precisely, presence and content function stalks for each gestalt are calculated. Steps (16)-(23) apply also to these resulting stalks.

5 RUBATO's MeloTopRUBETTE vs MeloRUBETTE

We compare our module, the MeloTopRUBETTE, with its first version as the RUBATO's MeloRUBETTE. There are many changes, mainly with respect to the outputs. We recall that RUBATO was first designed for experimenting a performance theory. That is why the output of the MeloRUBETTE, simulating a motivic analysis of a score, is meant to serve the performance Rubette. Therefore the unique output of the MeloRUBETTE is the resulting Mazzola note weights: To each note a real number is associated which corresponds to its 'motivic importance' within the whole score.

Our approach is different: we are interested in the topological model itself, and in its applications, for example in the performance theory. Since the validity of the latter model has not been tested yet, one first important feature is to have an easy access to each step of the program in order to adjust the parameters within the whole model. This is what we propose as major improvement of the MeloTopRUBETTE against the MeloRUBETTE: The interactive control of the ongoing computational process making possible to

improve and to extend the model "on the flight". In other words, this computational steering approach helps understanding and also modifying the mathematical model on motivic analysis of music. Also, the MeloTopRUBETTE does not hide any part of the topological model, on the contrary, it reveals all possible details about the motivic space of a score, which, on the other hand, is completely hidden by the MeloRUBETTE.

On the algorithm level, we introduced a significative more efficient calculation step: Calculations extend to classes (gestalts) of motives, thereby reducing considerably the amount of calculations. The reduced calculations are distance values for each pair of gestalts with same cardinality, as well as the evaluation of each quantification function (on the set of all gestalts instead of all motives). Because of the gained efficiency we could implement the evaluation of quantification functions at a collection of neighborhood radii compared to the evaluation at only one radius for MeloRUBETTE. As a consequence we can also calculate dimensions of quantification function spaces which is of course not possible with the MeloRUBETTE.

We also generalized the MeloRUBETTE by offering 9 shape types compared to 3 in the MeloRUBETTE. In particular, the "diastematic index" shape type from the MeloRUBETTE has been refined (as the contour shape type) to yield a motivic space, a topological structure which is impossible to define for the "diastematic index" shape type. Moreover, with this rich variety of shape types and the easy access to define new shape types, the contour similarity "theory" of the American Set Theory is a special case of our implementation, since their contour similarity concepts are, through our model, extended (see [6]) to a (motivic) topology on the space of all motives of a score, i.e. a structure in which a similarity concept between motives of different cardinalities is introduced.

There are also more possibilities of paradigmatic groups, as well as of distances. Moreover the quantification functions, which have been extended to gestalts, are implemented as their generalization. A special case is the Mazzola quantification functions as defined in section 4 and as implemented in the MeloRUBETTE.

Finally, the most significantly improvement of the MeloTopRUBETTE is the enrichment of the output: In the MeloRUBETTE, the unique output consisted of weights on notes of the analyzed score, whereas in the MeloTopRUBETTE, we propose weights on notes, weights on motives and on gestalts, the evaluated qualitative function necessary for the motivic evolution tree, the dimensions of real vector spaces of the presence, content, or

weight functions, dimensions of respective stalks of related function sheaves, and, as useful feature, a motif and gestalt information interactive window which outlines e.g. the motif's notes in the score, its weights at all radii, its gestalt (class of motives), its image in the motivic evolution tree, etc.

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