

AUTOMATIC MOTIVIC ANALYSIS INCLUDING MELODIC SIMILARITY FOR DIFFERENT CONTOUR CARDINALITIES: APPLICATION TO SCHUMANN'S *OF FOREIGN LANDS AND PEOPLE*

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ABSTRACT

This paper aims at presenting a topological model of motivic structure and analysis, and its application, via the implementation, to Schumann's *Of Foreign Lands and People* in *Scenes from Childhood*. Our immanent approach importantly includes the concept of contour similarity for different motif lengths making then possible to formalize the germinal motif (or leitmotif) concept. Based on motif, contour, gestalt, and motif similarity concepts, the crucial step in our mathematical model is indeed the introduction of neighborhoods of motives that include (similar) motives of different cardinalities and that yield a topological (T_0)-space on the set of all motives of a composition. In this space, the 'most dense' motif corresponds to the piece's 'germinal motif'. The model implementation (JAVA) constructs the spaces and in particular calculates, for each similarity threshold (neighborhood radius), the germinal motif; this is graphically represented in *motivic evolution trees*. The application to *Of Foreign Lands and People* briefly exemplifies our method.

1. INTRODUCTION

In the context of computer-aided analysis, melodic similarity is of high interest, though handled very often only partially and with difficulties. The particular case of measuring the similarity between two melodies of same length is normally easily managed. However, the extension of this concept to describe the 'similarity structure' of many melodies within a composition is much more complex and, without the use of mathematics, cannot be so easily handled. Especially when considering melodies of different cardinalities.

However it is clear that any reasonable model of a germinal motif, i.e. of that sequence of notes having a germinal function such as the *Leitmotif* in Beethoven's *Fifth symphony*, being heard literally or transformed all along the composition, necessitates the inclusion of melodies of different lengths into the method.

In this paper we present a topological model of motivic structure that includes the concept of contour similarity for different lengths, and an application to Schumann's

Of Foreign Lands and People in *Scenes from Childhood*. Our immanent approach favors melodic relationships below the musical surface as presented by Réti [11]. It is clear that by dealing with similar motives of different cardinalities it considerably enhance the complexity of the model and the computations. However, despite this complexity, the model has been entirely implemented (JAVA): see [4] as an improved (and complete model implementation) version of the software module MeloRUBETTE[®] in RUBATO[®] (see [9], [8]).

Indeed the complexity of calculations could be managed based on mathematical results considerably reducing calculations while keeping the immanent character of the approach: see [3], [4]. Investigations by use of the MeloRUBETTE[®] on Schumann's *Träumerei* [1], on Webern's *Variation für Klavier op. 27/2* [9], and on Bach's *Kunst der Fuge* [12] support the validity of our model. Also, it is important to mention that contour approaches based on *Set Theory* [10] (e.g. see [13]) can be redefined, generalized, and extended within our model: see [5], [3].

We will first shortly summarize our model of automatic motivic analysis based on a topological space of motives at section 2. It has a generic character and, importantly, allows the analyst to change perspectives (i.e. change topological parameters in the model) for the analysis. The model is based on the concepts of motif, contour, gestalt, motif similarity, and neighborhood of a motif including (similar) motives of different cardinalities. This yields a topological T_0 -space on a collection of motives in a composition.

The formalization of the *germinal motif* within the topological spaces is discussed at section 3. It basically corresponds to the motif with 'most dense' neighborhood for a given similarity threshold (radius). We shortly discuss the model implementation at section 4, and an application at section 5 to Schumann's *Of Foreign Lands and People* briefly exemplifies our method. In particular, we present a *motivic evolution tree (MET)* of this piece that graphically represents the germinal motif in function of the similarity threshold. The MET gives an overall motivic evolution spectrum, rather than just the identification of the germinal motif.

2. TOPOLOGICAL MODEL OF MOTIVIC STRUCTURE

In this section we shortly summarize the main concepts of our topological model. We restrict our attention to a minimal simplified setup in order to make the essentials clear; for details we refer the reader to [3], [7], [2].

Tones are parameterized by at least *onset* and *pitch* values and possibly by *duration*, *loudness*, *crescendo*, and *glissando* values. We introduce **motives** M as non-empty finite sets of tones: $M = \{m_1, \dots, m_n\}$ such that all onset values in M are different, and we set $\text{card}(M) = n$, the number of tones in M . Given a music composition S we consider a (finite) collection of motives in S that we denote $MOT(S)$. We impose that $MOT(S)$ satisfies the *Submotif Existence Axiom (SEA)*, that is every submotives of a motif in $MOT(S)$, down to a minimal cardinality n_{min} , is also in $MOT(S)$.

The **shape**¹ of a motif M is the image of M by a set mapping² $t : MOT(S) \rightarrow \Gamma_t$; for example, $\text{Com}(M) =$ the COM matrix (see e.g. [6]) of M , $\text{Rg}(M) =$ projection of M on the onset-pitch plane (i.e. representation of M by only its onset and pitch values), $\text{Dia}(M) =$ vector of consecutive pitch differences (i.e. of consecutive intervals), or $\text{India}(M) =$ vector of signs of consecutive pitch differences. These 4 shape examples are respectively called *COM-matrix*, *Rigid*, *Diastematic*, and *Index Diastematic* shape types.

We consider a group P action on shapes (i.e. on Γ_t) induced by a group action on the motives, e.g., the affine counterpoint paradigmatic group³ CP generated by translations in time, transpositions, inversions and retrogrades. The (t, P) -**gestalt of a motif** M is the set of all motives with shapes in the same P -orbit as $t(M)$: $\text{Ges}_t^P(M) := t^{-1}(P \cdot t(M))$. Gestalts conceptualize the identification of motives with their *imitations*. We also introduce pseudo-metrics d_n for shapes of cardinality n that we retract to motives: the **t -distance between two motives M and N with same cardinality n** is $d_t(M, N) := d_n(t(M), t(N))$, and their gestalt distance is

$$gd_t^P(M, N) := \inf_{p, q \in P} d_n(p \cdot t(M), q \cdot t(N)). \quad (1)$$

If P is a group of isometries, then gd_t^P is also a pseudo-metric. For example, the Euclidean distance on $\Gamma_t \subset \mathbb{R}^m$, for any shape type t , or e.g. the CSIM or C⁺SIM values [6] for the COM-matrix shape type, measure the distance (i.e. *contour similarity*) between motives M and N .

A crucial step in our model in order to formalize the contour similarity for different motif cardinalities, i.e. to

¹ The *shape* of a motif is a generalized concept for *contour*. Indeed, the usual contour of a motif corresponds to the *diastematic index shape* of a motif.

² Observe that the exact construction of the model is rather on the set MOT of all possible motives from which we take a finite collection $MOT(S)$ of motives in S . In particular, the domain of the shape mapping t is MOT . This is necessary for the consistency of the mathematical structure, but we put it on the side for this paper to privilege concepts to details (see [2]).

³ This terminology refers to Jean-Jacques Nattiez' *paradigmatic theme*.

formalize variations and transformations of motives, is the introduction of the **ϵ -neighborhood of a motif** M : it includes all motives N that contains a submotif N^* ϵ -similar to M . More precisely, given $\epsilon > 0$ we define $V_\epsilon^{t,d,P}(M) :=$

$$\{N \in MOT(S) | N^* \subset N \text{ s.t. } gd_t^P(N^*, M) < \epsilon\}. \quad (2)$$

If our setup (defined by fixing t , P , and d) fulfills the inheritance property ([2],[7]), the collection of all these neighborhoods forms a basis [2], [7] for a topology $\mathcal{T}_{t,P,d}$ on the set⁴ $MOT(S)$ of motives in S . In contrast with the other shape types we have to note that the *India* shape type does not satisfy the inheritance property since it has lost the 'global' motif structure information: see [2]. The topological space is called **motivic space of the composition** S .

The resulting T_0 -topological structure corresponds to the motivic structure of a composition, and the germinal motif (i.e. the *Leitmotif*) is formalized by the '*most dense*' motif given a (similarity threshold) radius ϵ . More precisely, because of the peculiar open neighborhoods of the space, we set the **presence of a motif M at radius ϵ** as $\text{pres}_\epsilon(M) :=$

$$\sum_{N \in MOT(S)} \frac{1}{2^{n-m}} \cdot \#\{N^* \subset N | gd_t^P(N^*, M) < \epsilon\}, \quad (3)$$

where $m = \text{card}(M)$ and $n = \text{card}(N)$, its **content** as $\text{con}_\epsilon(M) :=$

$$\sum_{N \in MOT(S)} \frac{1}{2^{m-n}} \cdot \#\{M^* \subset M | gd_t^P(M^*, N) < \epsilon\}, \quad (4)$$

and its **weight** as $\text{weight}_\epsilon(M) := \text{pres}_\epsilon(M) \cdot \text{con}_\epsilon(N)$. The **germinal motif**, given a similarity threshold ϵ , is formalized as the motif with largest weight in the composition. Finally, we represent graphically the germinal motif as function of the (similarity threshold) radius, representation that we call **Motivic Evolution Tree of S** . See [3], [5] for the detailed motivic space and motivic evolution tree of the main theme of Bach's *Art of Fugue*.

3. FORMALIZATION OF THE LEITMOTIF CONCEPT WITHIN OUR MODEL

Given⁵ a composition S to analyze we first take a collection $MOT(S)$ of sequences of notes, that we call *motives*, to be analyzed. It is at this step that we may use metric or harmonic structures in order to largely segment S . The *SEA* condition on $MOT(S)$ corresponds to the fact that the analyst naturally considers parts of motives for identifying similarity between motives. Abstraction of motives' features is formalized by the choice of a *shape type* and the *shape images of motives*.

The identification of imitations corresponds to *gestalts* of motives, and variations and transformations correspond

⁴ In the exact construction the topological space of a composition is defined as the relativization to $MOT(S)$ of the topology on MOT .

⁵ For detailed arguments about this section statements, see [7] and [3].

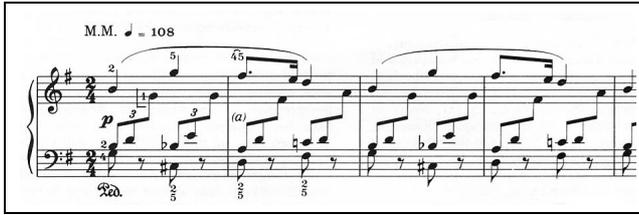


Figure 1. The beginning of Schumann's *Of Foreign Lands and People*, first piece of *Scenes of Childhood*

to being in the ϵ -neighborhood or containing in its ϵ -neighborhood, given a similarity threshold, i.e. a *radius*, ϵ . The difference between a variation and a transformation is that the latter admits a larger ϵ value. The obtained *topological space* for S corresponds to the motivic structure of S [3],[7]. The formalization of the germinal function of a motif, i.e. of being omnipresent in a composition, depends on the similarity threshold ϵ , and is done by measuring for each motif in S its *presence* and its *content*, summarized together as its *weight*.

Finally, given a composition S , a segmentation of S , a shape type t , a group P of imitations, a similarity measure d , and a similarity threshold ϵ , the germinal motif is the sequence of notes with *largest weight* for ϵ [3],[7]. The leitmotif of S for each ϵ is represented graphically in the *motivic evolution tree*.

We stress that this formalization of the germinal motif concept naturally corresponds to Rudolph Réti's immanent approach to motivic analysis, though after a precision on his fuzzy terminology [3],[7].

4. MODEL IMPLEMENTATION

This model was first partially implemented in 1996 by G. Mazzola and O. Zahorka [9],[8] as a module of the software RUBATO. As a major improvement in the JAVA implementation [4] by the author in 2002 we mention the totality of the (same but extended) model that is implemented and most importantly that is also unveiled with details through the rich diversity of the outputs. Also the computational efficiency has been greatly improved based on a mathematical theorem [3] asserting that the topological construction on motives can be restated directly on gestalts (motif classes) which considerably reduces computations. For example, from 355,299 2-to-5 note motives in Schumann's *Dreamery* there are 172 (*Com*, *CP*)-gestalts. At the time this paper is written all outputs of the implementation are still numerical, except for the motivic evolution trees that are implemented in Mathematica[®].

It is important to mention that the implementation keeps the generic character of the model. There are therefore many analytic parameters, such as the shape type, the admissible imitations or the similarity measure functions, to be set when analyzing a composition. It allows to have different perspectives on the composition (application of the Yoneda Lemma: see [7]).

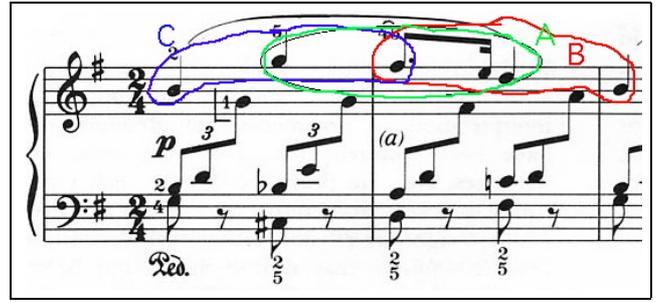


Figure 2. According to different topological settings and shape similarity thresholds, motives A: G-F#-E-D, B: F#-E-D-B, and C: B-G-F# are identified as germinal motives. See Figure 3 for the overall motivic evolution spectrum.

5. APPLICATION TO SCHUMANN'S OF FOREIGN LANDS AND PEOPLE

We present an application of our approach to Schumann's *Of Foreign Lands and People*, first piece of *Scenes of Childhood*. See Figure 1 for the first measures. Because of the limitation of the numerical outputs we only briefly comment the analysis.

We considered different segmentations for the analysis depending on the topological setting. For example, notes of the composition were split into 2 voices (upper and lower voices), and all 2-to-8 note motives with maximal span of 2.5 bars, were calculated (i.e. $MOT(S)$). We set the shape type t to *COM*-Matrix and diastematic types. Respectively the group P is set to the group Tr of translations in time and transpositions and to the counterpoint group CP . Table 1 shows for each motif cardinality the number of motives in $MOT(S)$ and the number of corresponding gestalts that were considered in the analysis. We used the relative Euclidean distance d_t . For example, the distance values varied from 0.167 and 22.0 for $t = Dia$ and $P = Tr$.

In Figure 3 we present the *motivic evolution tree* of the analysis with $t = Dia$. This represents the germinal motif (dark:highest weight and pale:2nd highest weight) evolving from top to bottom as the similarity threshold (neighborhood radius) gets larger. The horizontal axis is the motif cardinality. The pale lines between motives in the graphic correspond to the (transitive) submotif relation, therefore the linked motives are topologically close to each other. For example, at radius $\epsilon = 0.47$ the germinal motif is F#-E-D-B (4-40 in the tree) from the upper voice (see Figure 2). It has shape $(-2, -2, -3)$ and is *Tr*-imitated 12 times within our segmentation. At similarity thresholds from $\epsilon = 0.745$ to 0.9, the motif G-F#-E-D (4-20 in the tree) with shape $(-1, -2, -2)$ is the germinal motif (see Figure 2). Also, there is a clear descending melodic line constant through the tree, as expected with our choice of analysis parameters. With $t = Com$ and $P = CP$, the corresponding motivic evolution tree shows the germinal motif B-G-F# (see Figure 2) until $\epsilon = 0.221$.

