

Melodic Clustering Within Topological Spaces of Schumann's *Träumerei*

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Abstract

*This paper aims at extending a topological model of motivic analysis and structure in order to insert the concept of clustering. Our immanent approach includes the contour similarity for motives of different cardinalities making possible to formalize the concept of germinal motif. Based on motif, contour, gestalt, and motif similarity concepts, neighborhoods of motives are introduced and yield (T_0) -topological spaces. Given a collection X of motives to be organized in melodic clusters, we extract the clusters from the topological space by considering, within the relative space X , intersections of some sets corresponding to the formalization of motif variations within the model. The model extension was implemented and applied to Schumann's *Träumerei*. The resulting clusterings are successfully compared to the music theorist Repp's melodic segmentation of the piece (1992).*

1 Introduction

Melodic similarity is a topic of high interest in the contexts of computer-aided analysis and content-based music retrieval, as shown in recent works such as (11), (8), (9), and (7).

It involves measuring similarity between melodies, though the similarity between two melodies of different lengths is very often handled only partially and with difficulties. But it is clear that in computer-aided analysis, any reasonable model of a germinal motif, i.e. of that short melodies having a germinal function such as the opening motif in Beethoven's *Fifth symphony*, being heard literally or transformed all along the composition, necessitates the inclusion of melodies of different lengths into the method.

Our topological approach (2) to the modeling of motivic structure does include the concept of contour similarity for different lengths. This immanent approach favors melodic relationships below the musical surface as presented by Réti

(18), (19). By dealing with similar motives of different cardinalities it considerably enhance the complexity of the model and the computations. However, despite this complexity, the model has been entirely implemented (JAVA): see (5) as an improved (and complete model implementation) version of the software module MeloRUBETTE[®] in RUBATO[®] (see (15), (14)). Note that our approach does not restrict to monophonic music.

In our model, melodic segments are compared with one another in order to determine which melodic segments are germinal motives. Another analytical structure of importance is the melodic clustering, that is an organization of melodic segments into 'significant' categories.

In this paper we present our model extension to melodic clustering and an application to Robert Schumann's *Träumerei*, the seventh piece of *Kinderszenen*, op.15. The resulting clustering is compared to the music theorist Repp's melodic segmentation (17) as well as to the computer-generated clustering proposed by E. Cambouropoulos & G. Widmer (10). The results are extremely close to these reference clusterings. As significant consequence, it strongly contributes to the validation of our topological model. We add that investigations by use of the Melo-RUBETTE[®] on Schumann's *Träumerei* (1) (investigation in 2000), on Webern's *Variation für Klavier op. 27/2* (15), and on Bach's *Kunst der Fuge* (20) support the validity of our model. Also, contour approaches based on *Set Theory* (16) (e.g. see (21)) can be redefined, generalized, and extended within our model: see (6), (4).

In section 2, we first briefly summarize our model of automatic motivic analysis based on a topological space of motives. It has a generic character and allows the analyst to change perspectives (i.e. change topological parameters in the model) for the analysis. Our model is based on the concepts of motif, contour, gestalt, motif similarity, and neighborhood of a motif including (similar) motives of different cardinalities. This yields a topological T_0 -space on a collection of motives in a composition. We exemplify all concepts

of our model within Schumann’s *Träumerei*.

The extension of our model to melodic clustering is presented in section 3. It concerns the intersection of some sets that correspond to the formalization of motif variations. We shortly discuss the model implementation at section 4. In section 5, we apply our approach to Schumann’s *Träumerei* and compare the resulting clusterings (Tables 4 to 7) to the reference clusterings: Repp (17) (Table 1) and Cambouropoulos & Widmer (10) (Tables 2 - 3).

2 Motivic Space of a Composition

In this section we shortly summarize and exemplify in Schumann’s *Träumerei* the main concepts of our topological spaces of compositions. We restrict our attention to a minimal simplified setup in order to make the essentials clear; for details about the model and about its meaning in music, we refer the reader to (4), (13), (3).

Tones are parameterized by at least *onset* and *pitch* values and possibly by *duration*, *loudness*, *crescendo*, and *glissando* values. **Motives** M are non-empty finite sets of tones: $M = \{m_1, \dots, m_n\}$ such that all onset values in M are different, and we set $\text{card}(M) = n$, the number of tones in M . Given a music composition S we consider a (finite) collection of motives in S that we denote $MOT(S)$. We impose that $MOT(S)$ satisfies the *Submotif Existence Axiom (SEA)*, that is every sub-motives of a motif in $MOT(S)$, down to a minimal cardinality n_{\min} , is also in $MOT(S)$.

For example, if we take the following note gauge: value 60 for C_4 and value 1 for a semi-tone, value $3/4$ for the first score note C onset and value 1 for a bar duration, then motif 2 in Figure 1, denoted by M , can be represented by

$$M = \{(2.75, 76), (2.875, 74), (3, 72), (3.125, 77)\}.$$

For $MOT(S)$, we select all 28 motives from Figure 1, together with all their sub-motives down to cardinality 2, for a total of 1438 motives. Another example for $MOT(S)$ could be all motives from the soprano voice that are between 2 and 10 notes for which the first and last notes are at most 2 bars apart from one another. This would lead to a total of 237,736 motives.

The **shape**¹ of a motif M is the image of M by a set mapping² $t : MOT(S) \rightarrow \Gamma_t$; for example, $\text{Com}(M) =$ the COM matrix (see e.g. (12)) of M , $\text{Rg}(M) =$ projection of M on the onset-pitch plane (i.e. representation of M by only its onset and pitch values), $\text{Dia}(M) =$ vector of consecutive pitch differences (i.e. of consecutive intervals), or

¹The *shape* of a motif is a generalized concept for *contour*.

²Observe that the exact construction of the model is rather on the set MOT of all possible motives from which we take a finite collection $MOT(S)$ of motives in S ; see (3) for details.

$\text{El}(M) =$ vector of angles of consecutive onset-pitch note segments with the (horizontal) onset axis and of Euclidean lengths between consecutive notes, over the overall length. These 4 shape examples are respectively called *COM-matrix*, *Rigid*, *Diastematic*, and *Elastic* shape types.

For example, if we consider again $M = \text{motif 2}$ from Figure 1, we have

$$\text{Com}(M) = \begin{pmatrix} 0 & -1 & -1 & 1 \\ 1 & 0 & -1 & 1 \\ 1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 0 \end{pmatrix},$$

$\text{Rg}(M) = ((2.75, 76), (2.875, 74), (3, 72), (3.125, 77))$ ³, $\text{Dia} = (-2, -2, 5)$, and $\text{El}(M) = (-1.508, -1.508, 1.546, 0.222, 0.222, 0.555)$.

We consider a group P action on shapes (i.e. on Γ_t) induced by a group action on the motives, e.g., the affine counterpoint paradigmatic group⁴ CP generated by translations in time, transpositions, inversions and retrogrades. The (t, P) -**gestalt of a motif** M is the set of all motives with shapes in the same P -orbit as $t(M)$: $\text{Ges}_t^P(M) := t^{-1}(P \cdot t(M))$. Gestalts conceptualize the identification of motives with their *imitations*. The gestalt identification highly depends on the shape type.

For example, if we consider the group $P = \text{Tr}$ of transpositions and translations in time and the diastematic shape type, the motif M (motif 2) and motif 19 are identified together as having same gestalt since they have same diastematic shape: $\text{Dia}(\text{Motif 19}) = (-2, -2, 5)$. With same group but with the COM-matrix shape type, the G-F-E-A sub-motif composed of the 1st, 2nd, 3rd and 5th notes from motif 7, is also identified with motif 2. With the shape type $t = \text{Dia}$, the 1438 motives in $MOT(S)$ are regrouped in 382 gestalts for $P = \text{Tr}$ and in 322 gestalts for $P = CP$. For $t = \text{Com}$, the 1438 motives in $MOT(S)$ are regrouped in 164 gestalts for $P = \text{Tr}$ and in 123 gestalts for $P = CP$.

We introduce pseudo-metrics d_n for shapes of cardinality n that we retract to motives: the **t -distance between two motives M and N with same cardinality n** is $d_t(M, N) := d_n(t(M), t(N))$, and their gestalt distance is

$$gd_t^P(M, N) := \inf_{p, q \in P} d_n(p \cdot t(M), q \cdot t(N)). \quad (1)$$

If P is a group of isometries, then gd_t^P is also a pseudo-metric. For example, the relative Euclidean distance REd_t on $\Gamma_t \subset \mathbb{R}^m$, for any shape type t , or e.g. the CSIM or

³In this example, the rigid shape of the motif is almost the same as the motif itself. When other parameters are taken into account for representing notes, such as duration or loudness, the rigid shape is a very useful shape type.

⁴This terminology refers to Jean-Jacques Nattiez’ *paradigmatic theme*.

C^+ SIM values (12) for the COM-matrix shape type, measure the distance (i.e. *contour similarity*) between motives.

For example, If we take motif 3, *G-A-Bb-D*, in Figure 1, i.e. $N = \{(3.25, 67), (3.375, 69), (3.5, 70), (3.625, 74)\}$ with COM-matrix shape

$$Com(N) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 \\ -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & 0 \end{pmatrix},$$

$P = CP$ and $d_{Com} = REd$, then $d_{Com}(M, N) = 1.225$, and $gd_{Com}(M, N) = 0.354$ since the inverse of N minimizes the distance between the two gestalts, i.e. $gd_{Com}(M, N) = d_{Com}(M, inverse(N))$.

A crucial step in our model in order to formalize the contour similarity for different motif cardinalities, i.e. to formalize variations and transformations of motives, is the introduction of the ϵ -neighborhood of a motif M : it includes all motives N that contains a submotif N^* ϵ -similar to M . More precisely, given a $\epsilon > 0$, we define $V_\epsilon^{t,d,P}(M) :=$

$$\{N \in MOT(S) | N^* \subset N \text{ s.t. } gd_t^P(N^*, M) < \epsilon\}, \quad (2)$$

which we may simply denote $V_\epsilon(M)$.

For example, for $t = Com$, $P = Tr$, and $d = REd$, the 0.001-neighborhood of motif 2 includes motives 7 and 19, and its 0.354-neighborhood contains also motives 11, 15, and 24. At $\epsilon = 0.001$, all motives in the neighborhood are linked with a strict sub-motif relation with same gestalt as motif 2: motif 19 has same gestalt as motif 2; the 1st, 2nd, 3rd and 5th notes of motif 7 form a motif with same gestalt as motif 2. At $\epsilon = 0.354$, small variations are allowed: e.g. the four last notes of motif 24 form a motif with almost same gestalt as motif 2.

If our setup (defined by fixing t , P , and d) fulfills the inheritance property ((3),(13)), the collection of all these neighborhoods forms a basis (3), (13) for a topology $\mathcal{T}_{t,P,d}$ on the set⁵ $MOT(S)$ of motives in S . The topological space is called **motivic space of the composition S** .

We briefly mention that, in our model, the abstraction of motives' features is formalized by the choice of a *shape type* and the *shape images of motives*, the identification of imitations corresponds to *gestalts* of motives, and variations and transformations correspond to being in the ϵ -neighborhood or containing in its ϵ -neighborhood, given a similarity threshold, i.e. a *radius*, ϵ . The obtained *topological space* for S corresponds to the motivic structure of S (4),(13). The formalization of the germinal function of a motif, i.e. of being omnipresent in a composition, is commented in section 5.1.

⁵In the exact construction the topological space of a composition is defined as the relativization to $MOT(S)$ of the topology on MOT .

3 Clustering Within Motivic Spaces

We introduce the definition of a cluster in our topological spaces in order to extract the corresponding significant sets of motives as identified by music theorists in their motivic analysis of compositions. Given a collection $X \subset MOT(S)$ of motives and a similarity threshold $\epsilon > 0$, the ϵ -variation set of motif M in X is

$$Var_\epsilon^X(M) := \{N \in X | N \in V_\epsilon(M) \text{ or } M \in V_\epsilon(N)\}. \quad (3)$$

For each motif M , this set can be rewritten as

$$Var_\epsilon^X(M) = (V_\epsilon(M) \cup W_\epsilon(M)) \cap X, \quad (4)$$

where $W_\epsilon(M) = \{N \in MOT(S) | M \in V_\epsilon(N)\}$, a closed set in $MOT(S)$. The intersection with the set X corresponds to consider the relative subspace to X . In this subspace, if M has minimum cardinality, then $W_\epsilon(M) \subset V_\epsilon(M)$ and $Var_\epsilon^X(M)$ is an open set, and oppositely if M has maximum cardinality, $W_\epsilon(M) \supset V_\epsilon(M)$, and therefore $Var_\epsilon^X(M)$ is a closed set.

In order to model clustering approaches, we introduce an additional set C_M intersection with the variations. We want this intersection set to depend on the cardinality of the motif M . For example, we require that motives in the variation set of M should have a cardinality of at least 70% of the cardinality of M . This can be formalized as

$$C_M = \{N \in X | \frac{\min(card(M), card(N))}{\max(card(M), card(N))} \geq 70\%\}. \quad (5)$$

Another possibility is to consider motives with cardinality at most off by one to the cardinality of M . In that case, we would have:

$$C_M = \{N \in X | card(M) - 1 \leq card(N) \leq card(M) + 1\}. \quad (6)$$

Finally, we introduce the ϵ -cluster $Cluster_\epsilon^X(M)$ of motif M in X (and with respect to C_M) as

$$Cluster_\epsilon^X(M) := Var_\epsilon^X(M) \cap C_M \quad (7)$$

We call X the **clustering set**. Given a set of motives X and a cardinality condition for the C_M sets, **clustering the collection X of motives** corresponds to calculate all the ϵ -clusters $Cluster_\epsilon^X(M)$, i.e. for all motives $M \in X$ and all similarity thresholds $\epsilon > 0$.

4 Model Implementation

The initial model was first partially implemented in 1996 by G. Mazzola and O. Zahorka (15),(14) as a module of the

software RUBATO. It was completely reimplemented in JAVA in 2002 by the author for which the major improvements are the rich diversity of the outputs unveiling all details of the topological spaces and a significant enhancement of computational efficiency. Output of the implementation is mostly numerical, except for the motivic evolution trees (see Figure 2) that are implemented in Mathematica[©]. The clustering extension of the model has just been appended (2006) to our program.

The implementation keeps the generic character of the model. There are therefore many analytic parameters, such as the shape type, the admissible imitations or the similarity measure functions, that need to be set when analyzing a composition. It allows to have different perspectives on the composition (application of the Yoneda Lemma: see (13)), as exemplified in the next section.

5 Application to Schumann's *Träumerei*

We construct topological spaces of Schumann's *Träumerei* from which we extract clusterings of the segmented soprano voice (see Figure 1).



Figure 1: The soprano voice of Schumann's *Träumerei* together with the melodic/rhythmic segmentation proposed by Bruno Repp (13).

We compare our clustering results with the melodic/ rhythmic segmentation suggested by the music theorist B. Repp (17) (see Table 1) and with the computer-generated clustering done by E. Cambouropoulos & G. Widmer (10) through what they call an *exact matching* (see Table 2) and a *near-*

exact matching (see Table 3) approaches⁶. Our main goal is to obtain a clustering close to the segmentation from Repp and to measure up our automatic procedure with the above-mentioned computer approaches.

Clustering tables should read as follows. Each cell in the table corresponds, in a consistent manner, to a motif (called a '*melodic gesture*' by Repp) in Figure 1. The very left column indicates the phrase structure of the *Träumerei* containing two main phrases, A and B, that appear in some variations (Ai and Bi). The letter symbols correspond to clustering labels (accordingly to (10)). An empty cell in a table corresponds to a monadic category.

A1	a	b	c	d	e	
B1	a	b		f	g	
B2	a	b		f	g	
B3	a	b		f	g	
A1	a	b	c	d	e	
A2	a	b	c	d	e	h

Table 1 Melodic segmentation of Schumann's *Träumerei* soprano voice according to the music theorist B. Repp (17).

A1	a	b	c	c	e
B1	a	b			
B2	a	b		f	
B3	a	b		f	
A1	a	b	c	c	e
A2	a	b		c	c

Table 2 Melodic clustering of Schumann's *Träumerei* soprano voice generated by an *exact matching* computer approach by E. Cambouropoulos and G. Widmer (10). Empty spaces correspond to monadic categories.

A1	a	b	c	d	e	
B1	a	b				
B2	a	b		f	g	
B3	a	b		f	g	
A1	a	b	c	d	e	
A2	a	b	c	d	d	d,e

Table 3 Melodic clustering of Schumann's *Träumerei* soprano voice generated by a *near-exact matching* computer approach by E. Cambouropoulos and G. Widmer (10). Empty spaces correspond to monadic categories.

⁶Both approaches were applied on the surface level and on a reduced level, concluding each with the same clustering.

In order to compare our results, we too restrict the composition to the soprano voice of the *Träumerei* and consider all 28 motives suggested by Repp: see Figure 1. Motivic spaces are constructed for which the set $MOT(S)$ of motives consists of the 28 motives together with all their sub-motives down to cardinality 2. This leads to 1438 motives in $MOT(S)$.

In our analysis, four different shape types are considered: $t = Rg, Dia, El,$ and Com with two different paradigmatic groups: $P = Tr$ and CP . We select the relative Euclidean metric for the distance function, i.e. $d_{t,n} = REd_{t,n}$. The clustering set X strictly contains the 28 motives from Figure 1. As in (10), we impose in the clustering a cardinality ratio restriction of 70% and fix a similarity threshold for each shape type. Tables 4 to 7 show the resulting clusterings embedded in their corresponding topological spaces.

A1	a	b	c	d	d,e
B1	a	b		f	z
B2	a	b		f	g
B3	a	b		f	g
A1	a	b	c	d	d,e
A2	a	b	c	d	d d,a,z

Table 4 Melodic clustering within the topological space of Schumann’s *Träumerei* soprano voice with the following parameters: $t = Rg, P = Tr,$ and $d_t = REd_t,$ and the similarity threshold $\epsilon_{max} = 0.43$.

Within the topological space obtained by selecting the rigid shape type, the resulting clustering (Table 4) is very close to Repp’s melodic segmentation (Table 1). The four clusters a,b, c and f are successfully regrouped in the space. Cluster g is missing one motif. In the space, cluster e is regrouped with cluster d since motives 4 and 21 both have a sub-motif of cardinality 4 that are at 0.108-distant to motives 3, 20, 25 and 26 forming cluster d. We mention that Repp claims (17): “[motif 4, 21 ,26], reinforced by shorter secondary [motivic gestures] in the bass voice, is similar to [motif 3, 20, 25], ...”. In our case, this similarity is encapsulated within the topological space.

Indeed, as for the motif 26 being identified with cluster d, it is clear that our approach will never separate it from motif 25 since motif 26 is a strict repetition of 25. That is also the case in Cambouropoulos & Widmer’s approach. As they argue, the difference may come from a more global structure (see A_1, A_2, A_3 -parts) taken into account by Repp which, in our motivic spaces, cannot be detected. In other words, in this case of motives 25 and 26, there is a strict identification, a relation that is stronger than a plain similarity.

Cluster d also includes the last motif h (in Table 1) since motif 27 has same gestalt as the 3-first-note sub-motives of

motives 3, 20, 25 and 26. It is not regrouped with motives 4 and 21 since the 70% cardinality ratio is not satisfied. However, it is also linked to cluster a and to motif 8. Its rich association with 3 different clusters could be an argument for being a cluster on its own, as suggested by Repp.

Compared with the computer-generated clustering in Table 3, we gained in the identification of motif 7 with cluster f. However the regrouping of clusters d and e is not as close to Repp’s segmentation as does the near-exact matching approach.

A1	a	b	c	d	e
B1	a	b'		f	
B2	a	b		f	
B3	a	b		f	
A1	a	b	c	d	e
A2	a	b'	c	d	d d

Table 5 Melodic clustering within the topological space of Schumann’s *Träumerei* soprano voice with the following parameters: $t = Dia, P = Tr,$ and $d_t = REd_t,$ and $\epsilon_{max} = 0.471$. The empty entries correspond to monadic categories.

The motivic structure inside the topological space built by fixing $t = Dia$ gives again a clustering (see Table 5) very close to Repp’s melodic segmentation. The principal difference with the previous clustering is that clusters d and e are separate but cluster g is no more recognized. Also, motif 27 is only part of cluster d, making then impossible to distinguish it from the other clusters. The last observation is the loss of identification of cluster b into two clusters (b and b’ in the table).

A1	a	b	c	d	d,e
B1	a	b			
B2	a	b		f	g
B3	a	b		f	g
A1	a	b	c	d	d,e
A2	a	b	c,f,g	d	d d,e

Table 6 Melodic clustering within the topological space of Schumann’s *Träumerei* soprano voice with the following parameters: $t = Com, P = Tr,$ and $d_t = REd_t,$ and $\epsilon_{max} = 0.606$. The empty entries correspond to monadic categories.

With respect to the topological space with the comparison matrix shape type, $t = Com,$ and similarity threshold $\epsilon_{max} = 0.606,$ the clustering (see Table 6) is similar to the one for rigid shape type (see Table 4). The clusters a, b, and c are successfully identified, cluster g is again partially identified (motif 8 being on its own), and clusters d and e are

regrouped together. In this motivic space, motif 7 is not however identified with cluster f. Also, motif 24 gets identified with clusters c,f and g, since only the diastematic movement is taken into account in this topological space.

A1	a	b	c	d	d,e
B1	a	b			g'
B2	a	b	f		g,g'
B3	a	b	f		g,g''
A1	a	b	c	d	d,e
A2	a	b	c,f,g''	d	d d

Table 7 Melodic clustering within the topological Space of Schumann’s *Träumerei* soprano voice with the following parameters: $t = El$, $P = Tr$, and $d_t = REd_t$, and $\epsilon_{max} = 0.4$. The empty entry corresponds to a monadic categorie.

The clustering (see Table 7) embedded in the motivic space for the elastic shape type is similar to the previous one (Table 6). The main difference in this case is the identification of cluster g' and the motif 27 is being strictly in cluster d as in Table 5.

In overall, the clustering within all four topological spaces is very close to Repp’s categorization (Table 1). Clusters a, b, and c are clearly well identified. Clusters d and e are embedded with one another. This relates to Repp’s similarity association (see the quote below Table 4) between the two clusters. Finally, clusters f and g are only partially identified. Our automatic clustering compares well with the computer approach (Tables 2 and 3). In particular, the latter does not better identify clusters f and g.

5.1 Further Analytical Structure

First, we briefly mention that we can introduce, on the motivic space of a composition S , a weight function (4),(13) that measures the germinal significance of motives with respect to a (similarity threshold) radius. It then results in a graphical representation of the germinal motif in function of the similarity threshold, called *Motivic Evolution Tree (MET)* of S . See (4), (6) for the detailed motivic space and MET of the main theme of Bach’s *Art of Fugue*. See also (2) for the MET analysis of Schumann’s *Von Fremden Ländern und Menschen*.

Figure 2 shows an example of a MET for the *Träumerei*. It is not the purpose of this paper to present a thorough analysis of such METs (for each motivic space). We however mention that such an analysis together with the clustering does provide a rich and diverse melodic structure description of the composition. We intend to study in the future such METs and weight functions restricted to the relative topological space

X (the clustering set), and to compare them with their global equivalent (the actual weights and METs).

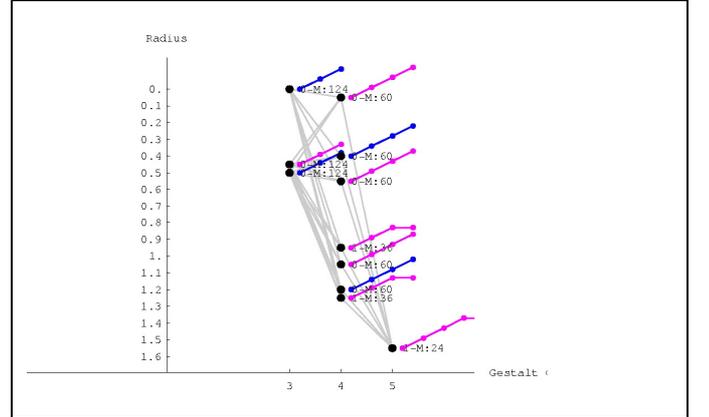


Figure 2: The motivic evolution tree of the *Träumerei* for the topological space defined with $t = Com$, $P = Id$, and $d_{Com} = CSIM$. From top to bottom, it shows the germinal motif representatives for a given similarity threshold (vertical axis). Black motives are most significant motives whereas gray motives are second most significant motives.

6 Conclusion

In this paper, we presented a model of melodic clustering as an extension of our topological model of motivic analysis and structure. In topological spaces of compositions, melodic clustering is straightforwardly achieved by introducing the intersection of some sets related to the formalization of motif variation. Given a composition and the analysis parameters (such as the contour type and the imitation group), topological spaces are constructed, and for a given surface segmentation, all its melodic clusters in function of the similarity threshold (basis neighborhood radii in the space) are calculated.

The application, through our computer implementation, to Schumann’s *Träumerei* (soprano voice) exemplified the model, and its resulting clusterings, within diverse topological spaces, showed a classification that is accurate when taking the melodic segmentation of B. Repp (17) as reference. Also, our automatic clusterings compared well with the clustering generated by another computer approach (*near-exact matching* (10)).

The clustering results strongly contributes to the validation of our topological model of motivic structure.

It is next intended to produce a thorough analysis of the *Träumerei*, considering not only the soprano voice but also

the whole composition, by comparing different local and global motivic evolution trees, as well as calculating and comparing the clustering of the whole composition (4 voices) to the clustering suggested by B. Repp in (17).

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