

# BROCK UNIVERSITY MATHEMATICS MODULES

## 11A2.6 Determining a Formula for a Quadratic Function

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### WWW

- What it is: Determining the formula for a quadratic function when given certain information such as its zeros, axis of symmetry, vertex, etc.
- Why you need it: It is important to be able to construct the formula for a quadratic function to better visualize, understand and analyze the relationship.
- When to use it: The methods we will discuss are used to analyze functions that represent real-life situations.

### PREREQUISITES

Before you tackle this module, make sure you have completed these modules:

**11A2.1 Determining Zeros for a Quadratic Function, 11A2.2 Factoring Quadratic Expressions, 11A2.3 Completing the Square, 11A2.4 Maximum or Minimum Values for Quadratic Functions, Graphical Meaning of Coefficients.**

### WARMUP

Before you tackle this module, make sure you can solve the following exercises. If you have difficulties, please review the appropriate prerequisite modules.

(Answers below.)

1. Factor each quadratic function.  
(a)  $y = x^2 - 2x + 1$  (b)  $h = 2t^2 - 3t - 9$  (c)  $f = 121 - 4x^2$  (d)  $g = 7x^2 + 14x - 21$
2. Find the zeros of each function in Question 1.
3. Determine an equation for the axis of symmetry for each function.  
(a)  $y = x^2$  (b)  $h = 3(t + 4)^2$  (c)  $d = -(t - 9)^2 + 403$  (d)  $f = -x^2 - 37$
4. Determine the coordinates of the vertex for each function in Question 3.
5. Determine the coordinates of the vertex for each function in Question 1.

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Answers: 1.(a)  $y = (x - 1)^2$  (b)  $h = (2t + 3)(t - 3)$  (c)  $f = (11 - 2x)(11 + 2x)$  (d)  $g = 7(x + 3)(x - 1)$   
2.(a)  $x = 1$  (b)  $t = -3/2, t = 3$  (c)  $x = 11/2, x = -11/2$  (d)  $x = -3, x = 1$  3.(a)  $x = 0$  (b)  $t = -4$   
(c)  $t = 9$  (d)  $x = 0$  4.(a) (0, 0) (b) (-4, 0) (c) (9, 403) (d) (0, -37) 5.(a) (1, 0) (b) (3/4, -81/8) (c) (0, 121) (d) (-1, -28)

## Introduction

In previous modules, you learned how to manipulate quadratic functions to show important information about their graphs. For instance, you learned how to rearrange a quadratic expression into vertex form to find the maximum or minimum values of a parabola, and to factor a quadratic expression to determine its zeros.

In real-world applications you often do not know the formula that may model a certain situation at first. Instead, you must use various facts to construct the formula yourself; the goal of this module is to teach you how to take basic information about a graph, such as its vertex, axis of symmetry, roots, and/or other points, and determine a formula for the graph. So, let's get started!

### FOCUS QUESTION

To help you understand an important aspect of this lesson, focus your attention on this question, which will be answered towards the end of the lesson.

**Krissa is on the competitive diving team at her school. When jumping from a 10 metre diving board, she reaches her highest point after 1 second and hits the surface of the water after 3 seconds have passed. Determine a formula for the quadratic function that models Krissa's height  $h$  in metres above the surface of the water after  $t$  seconds?**

**In order to perform a new diving trick, Krissa's instructor tells her that she needs at least 13 metres of space above the surface of the water. Using the information given, will Krissa be able to perform the new trick?**

Let's begin by reviewing some key forms of a quadratic function.

### DEFINITION

The standard form of a quadratic function is  $y = ax^2 + bx + c$ , where  $a$ ,  $b$  and  $c$  are real numbers.

In this form, it may be difficult to completely visualize the graph of the parabola, because we may not be able to easily deduce any significant points (i.e. the vertex or the zeros) just by looking at the formula; to do this we have to rearrange the formula by factoring or completing the square.

If it can be factored, the formula for a quadratic function can take on many different forms.

### DEFINITION

If a quadratic function has two real zeros,  $x = r$  and  $x = s$ , it can be written in the form  $y = a(x - r)(x - s)$  where  $a$ ,  $r$ , and  $s$  are real numbers. If the zeros are equal, so that  $r = s$ , then the function can be written in the form  $y = a(x - r)^2$ .

By completing the square, we obtain the vertex form of a quadratic function.

## DEFINITION

The vertex form of a quadratic function is  $y = a(x - p)^2 + q$ , where  $a$ ,  $p$ , and  $q$  are real numbers. The coordinates of the vertex are given by  $(p, q)$ .

In most questions that ask you to determine a quadratic function, you will be provided with information that will represent some, but not all of the variables described above. You will have to use the values you are given to solve for the variable(s) you do not know, but are necessary in determining the quadratic function.

The first step in solving these kinds of problems is deciding whether you want to use the factored or vertex form of a quadratic function as your starting point. Most of the time it is clear which form will work best because of the type of information you are given. However, sometimes this step can be a little tricky.

Let's go through a few examples to become familiar with the kinds of questions you may encounter.

## EXAMPLE 1

The graph of a quadratic function passes through point  $A(1, 1)$  and has a vertex at  $(3, -11)$ . Find a formula that represents the quadratic function.

## SOLUTION

Because we have been given the coordinates of the vertex, let's use the vertex form of a quadratic function,  $y = a(x - p)^2 + q$ . Because  $p = 3$  and  $q = -11$ , in our case this formula takes the form  $y = a(x - 3)^2 - 11$ . To determine the value of  $a$ , substitute the known point  $(1, 1)$  into the expression to obtain

$$\begin{aligned}y &= a(x - p)^2 + q \\y &= a[x - 3]^2 + (-11) \\1 &= a(1 - 3)^2 - 11 \\1 + 11 &= 4a \\12/4 &= a \\a &= 3\end{aligned}$$

Therefore, a formula for the quadratic function is  $y = 3(x - 3)^2 - 11$ .

## EXAMPLE 2

A quadratic function has zeros at  $x = 1$  and  $x = 3/4$ . Another point on the graph is  $G(3, 15)$ . Determine a formula for this function.

## SOLUTION

We have been given the zeros of the function, which suggests that we use the factored form of a quadratic function:  $y = a(x - r)(x - s)$ . The zeros are  $r = 1$  and  $s = 3/4$ , so the function has the form  $y = a(x - 1)(x - 3/4)$ . Using the coordinates of the point  $G$ , we can determine the value of  $a$ :

$$\begin{aligned}
 y &= a(x - 1) \left( x - \frac{3}{4} \right) \\
 15 &= a(3 - 1) \left( 3 - \frac{3}{4} \right) \\
 &= a(2) \frac{9}{4} \\
 &= \frac{9}{2} a \\
 a &= \frac{10}{3}
 \end{aligned}$$

Therefore, a formula for the quadratic function is

$$y = \frac{10}{3}(x - 1)(x - 3/4)$$

### EXAMPLE 3

A quadratic function has only one zero at  $x = -8$ . If the graph passes through the point  $C(-10, -20)$ , determine a formula for the function.

### SOLUTION

Recall that if a parabola has only one zero, this point must also be the vertex. I will use the vertex form in my solution, but if I had chosen to use the factored form as my starting point, I would have reached the same result.

Substituting the information about the zero, we get:

$$\begin{aligned}
 y &= a(x - p)^2 + q \\
 &= a(x - (-8))^2 + 0 \\
 y &= a(x + 8)^2
 \end{aligned}$$

Now substitute the coordinates of the point  $C$  to determine the value of  $a$ :

$$\begin{aligned}
 -20 &= a(-10 + 8)^2 \\
 -20 &= a(-2)^2 \\
 -20 &= 4a \\
 a &= -\frac{20}{4} \\
 a &= -5
 \end{aligned}$$

Therefore, a formula for the quadratic function is  $y = -5(x + 8)^2$ .

#### EXAMPLE 4

The graph of a quadratic function opens up and is stretched vertically (compared to the graph of  $y = x^2$ ) by a factor of 10. The graph passes through the point  $D(1, 120)$  and its vertex is at  $x = 3$ . Determine a formula for the parabola.

#### SOLUTION

Since we do not know any information about the zeros of the function, but we are given information about its vertex, we will use vertex form.

Recall: To say that a graph is stretched or compressed vertically means that the coefficient of the  $x^2$  term is not 1. The coefficient  $a$  indicates the stretch or compression factor for a quadratic function. Thus,  $|a| = 10$ . Because the graph opens up, we can conclude that  $a = 10$ , not  $a = -10$ . Furthermore, because the vertex is at  $x = 3$ , we know that in vertex form the value of  $p$  is 3. That is, the function has the form  $y = a(x - p)^2 + q = 10(x - 3)^2 + q$ .

To determine the value of  $q$ , substitute the coordinates of the point  $D$  into the formula and solve for  $q$ :

$$\begin{aligned}y &= 10(x - 3)^2 + q \\120 &= (10)[(1) - (3)]^2 + q \\120 &= (10)(-2)^2 + q \\120 &= (10)(4) + q \\120 &= 40 + q \\120 - 40 &= q \\q &= 80\end{aligned}$$

Therefore, a formula for the quadratic function is  $y = 10(x - 3)^2 + 80$ .

#### KEY IDEA

The examples we have discussed so far give us some important hints about which form is easier to use as a starting point for finding a formula for a quadratic function. If we are given information about the vertex of the parabola, it is likely easiest to use the vertex form of a quadratic function. Similarly, if you know both zeros of a quadratic function and another point through which the graph passes, it is likely easiest to start by using the factored form of a quadratic function. However, Example 3 showed us that sometimes we can use either of the basic forms (for instance, if we are told that a function only has one zero) and it is up to us to choose which method we find easier.

Notice that before substituting the known values into a formula, we wrote out its general form. In practicing mathematics it is a good idea to adopt this habit because it helps you to remember important formulas.

Now it's time for you to try out some questions on your own:

## PRACTICE

(Answers below.)

- Find a formula for each quadratic function given its vertex  $(p, q)$  and a point  $(x, y)$  that its graph passes through.
  - $(p, q) = (-7, 2), (x, y) = (-4, 20)$
  - $(p, q) = (1/2, -13), (x, y) = (-1, -16)$
  - $(p, q) = (-4, -16), (x, y) = (-11, 33)$
- Find a formula for the quadratic function that passes through  $F(2, 3)$  and has zeros at
  - $x = 4$  and  $x = -2$
  - $x = -1/7$  and  $x = 12$
  - $x = 3$
- Find a formula for the quadratic function that
  - opens down, is compressed vertically by a factor of 2, passes through  $G(-1, -1)$  and has its vertex at  $x = 5$ .
  - opens up, is stretched vertically by a factor of 2, passes through the origin and has its vertex at  $x = -7$ .
  - opens down, is not compressed or stretched, has a  $y$ -intercept of  $-4$  and has its vertex at  $x = 9$ .

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Answers: 1.(a)  $y = 2(x+7)^2 + 2$  (b)  $y = -4/3(x-1/2)^2 - 13$  (c)  $y = (x+4)^2 - 16$  2.(a)  $y = -8/9(x-4)(x+2)$   
(b)  $y = -1/50(7x+1)(x-12)$  (c)  $y = 3(x-3)^2$  3.(a)  $y = -1/2(x-5)^2 + 17$  (b)  $y = 2(x+7)^2 - 98$  (c)  
 $y = -(x-9)^2 + 77$

In all of the examples and exercises so far, we have only encountered questions that ask us to find ONE unknown value before being able to construct the unique quadratic formula we are looking for. So, what do we do if we have TWO unknown values? And what happens if our solution is not unique? Let's consider the following two examples:

### EXAMPLE 5

The graph of a quadratic function passes through the points  $A(-1, -5)$  and  $B(8, 4)$ . If the axis of symmetry is  $x = 2$ , determine a formula that models this quadratic relation.

### SOLUTION

First, we notice that we have been given the  $x$ -coordinate of the vertex of the parabola. Therefore, we will consider the formula  $y = a(x-p)^2 + q$  as our starting point. We can immediately deduce that  $p = 2$ , which means the formula is  $y = a(x-2)^2 + q$ . This leaves us with two unknowns to determine,  $a$  and  $q$ . Because we have been given two separate points through which the graph passes, we are able to create two equations involving the two unknowns  $a$  and  $q$ , allowing us to solve the problem. Therefore, substitute the coordinates of each point into the formula, as follows.

Substitute  $x = -1$ ,  $y = -5$  to get:

$$\begin{aligned}y &= a(x - 2)^2 + q \\-5 &= a[(-1) - (2)]^2 + q \\-5 &= 9a + q\end{aligned}$$

Substitute  $x = 8$ ,  $y = 4$  to get:

$$\begin{aligned}y &= a(x - 2)^2 + q \\4 &= a[(8) - (2)]^2 + q \\4 &= 36a + q\end{aligned}$$

Now we can treat these two equations as a system of linear equations and use the method of substitution or elimination to solve for  $a$  and  $b$ . Since the  $q$  term in each equation has a coefficient of 1, we will use elimination to solve. First subtract the second equation from the first equation to get

$$-9 = -27a$$

and therefore  $a = -9 / -27 = 1/3$ . Substituting this value for  $a$  in the second equation<sup>a</sup> allows us to determine the value of  $q$ :

$$\begin{aligned}4 &= 36a + q \\4 &= 36\left(\frac{1}{3}\right) + q \\4 &= 12 + q \\q &= 4 - 12 \\q &= -8\end{aligned}$$

Therefore, an equation for the quadratic function is  $y = \frac{1}{3}(x - 2) - 8$ .

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<sup>a</sup>Substituting the value into the first equation would work just as well.

## KEY IDEA

If we know the coordinates of two points on a parabola and an equation for its axis of symmetry, we can determine a formula by constructing and solving a system of two linear equations. In this case, our task will be to determine values for  $a$  and  $q$ .

## COUNTER EXAMPLES

The graph of a quadratic function passes through the points  $K(-7, 8)$  and  $L(1, 8)$  and has its axis of symmetry at  $x = -3$ . Determine a formula for the function.

If we follow the structure of the previous example, we can start by observing that  $p = -3$ . Then, we can continue by obtaining two equations satisfied by  $a$  and  $q$ , as follows.

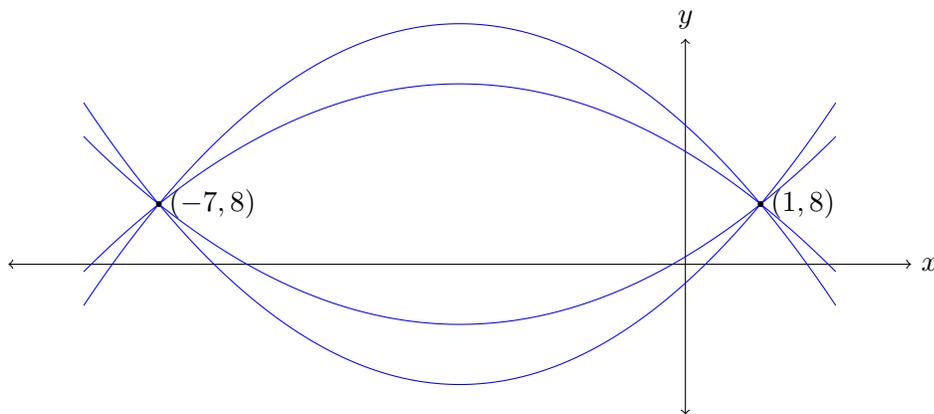
Because the point  $K(-7, 8)$  lies on the parabola,

$$\begin{aligned}y &= a(x - p)^2 + q \\8 &= a[(-7) - (-3)]^2 + q \\8 &= a[-7 + 3]^2 + q \\8 &= 16a + q\end{aligned}$$

Because the point  $L(1, 8)$  lies on the parabola,

$$\begin{aligned}y &= a(x - p)^2 + q \\8 &= a[(1) - (-3)]^2 + q \\8 &= a[1 + 3]^2 + q \\8 &= 16a + q\end{aligned}$$

Immediately, we realize that our equations are exactly the same! This means that there are an infinite number of solutions to the system. Therefore, there is not enough information provided to determine a single, unique solution for this problem. The figure shows some examples of parabolas that satisfy the conditions given in the question. (Note that the scales are different on the  $x$ -axis and  $y$ -axis.)



### EXAMPLE 6

A quadratic function has a zero at  $x = 5$  and the  $y$ -coordinate of its vertex is  $-32$ . The graph of the function opens up and is stretched vertically by a factor of 2. Determine a formula for the function.

### SOLUTION

Since we are given information about the vertex, let's use the vertex form of a quadratic function,  $y = a(x - p)^2 + q$ . Because the  $y$ -coordinate of the vertex is  $-32$ , this means that  $q = -32$ . The vertical stretch by a factor of 2 means that  $a = 2$ . This means we only have to determine the value of  $p$ ; we can use the last piece of information given, that there is a zero at  $x = 5$ . This means that when  $x = 5$ ,  $y = 0$ . Let's input the known quantities and solve for  $p$ :

$$\begin{aligned}
y &= a(x - p)^2 + q \\
y &= 2(x - p)^2 - 32 \\
0 &= 2(5 - p)^2 - 32 \\
0 &= (5 - p)^2 - 16 \\
0 &= (5 - p)^2 - 4^2 \\
0 &= (5 - p - 4)(5 - p + 4) \quad (\text{difference of squares}) \\
0 &= (1 - p)(9 - p)
\end{aligned}$$

Therefore,  $p = 9$  or  $p = 1$ . This means that there are two possibilities for our quadratic function:  $y = 2(x - 9)^2 - 32$  or  $y = 2(x - 1)^2 - 32$ .

Now you try!

### PRACTICE

(Answers below.)

4. Find a formula for the quadratic function that passes through the two given points and has an axis of symmetry with the given equation.
  - (a)  $A(0, 3)$ ,  $B(4, -9)$ ,  $x = -1$
  - (b)  $G(-2, 20)$ ,  $H(-11, -70)$ ,  $x = -4$
  - (c)  $E(7, 2)$ ,  $F(-2, -61)$ ,  $x = 3$
5. Find a formula for the quadratic function that satisfies the given properties.
  - (a) opens up, is stretched vertically by a factor of 4, passes through the origin and has its vertex at  $y = -4$
  - (b) opens up, is not stretched or compressed, passes through  $J(3, 11)$  and has its vertex at  $y = -14$
  - (c) opens down, is compressed by a factor of 4, has an  $x$ -intercept of 6 and has its vertex at  $y = 1$

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Answers: 4.(a)  $y = -\frac{1}{2}(x+1)^2 + 7/2$  (b)  $y = -2(x+4)^2 + 28$  (c)  $y = -7(x-3)^2 + 114$  5.(a)  $y = 4(x-1)^2 - 4$  and  $y = 4(x+1)^2 - 4$  (b)  $y = (x-8)^2 - 14$  and  $y = (x+2)^2 - 14$  (c)  $y = -\frac{1}{4}(x-8)^2 + 1$  and  $y = -\frac{1}{4}(x-4)^2 + 1$

So far, all of the questions we have answered have been strictly algebraic; the purpose of them is to learn the basic mechanics that are necessary in being able to solve word problems such as the focus question I stated at the beginning of this module. Let's apply what we've learned to find a formula for a quadratic function that models a real-life situation.

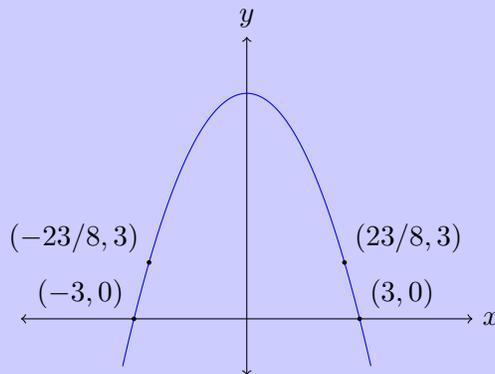
### EXAMPLE 7

Nicole spots a very large parabolic fountain in the park and wonders how high above the ground it reaches. She remembers how to find the equations of quadratic functions and knows that if she gathers some information, she will be able to estimate the height of the fountain. She measures that it is 6 feet wide at the base and 5 feet and 9 inches wide at a height of 3 feet above the surface of the water. Determine a formula for the quadratic function that represents the height  $h$  of the water in feet above the pond in which the fountain sits versus the distance  $x$  in feet away from the centre of the fountain.

### SOLUTION

We begin by drawing a diagram and labelling all of the information we have been given. Notice that part of drawing the diagram involves choosing where to place the parabola relative to a coordinate system. Also note that the  $x$ -axis and  $y$ -axis have different scales.

Because 5 feet 9 inches equals  $23/4$  feet, the points  $(23/8, 3)$  and  $(-23/8, 3)$  are on the graph.



We know four different points on our graph, including the zeros. Therefore, we will use the factored form of a quadratic function,  $h = a(x - r)(x - s)$ . Because the zeros are  $-3$  and  $3$ , we know that  $r = -3$  and  $s = 3$ . Thus,  $h = a(x - (-3))(x - 3) = a(x + 3)(x - 3)$ . Now substitute one of the other two known points (let's use  $(23/8, 3)$ , OK?) into the formula and solve for  $a$ :

$$\begin{aligned}h &= a(x + 3)(x - 3) \\3 &= a\left(\frac{23}{8} + 3\right)\left(\frac{23}{8} - 3\right) \\3 &= a\left(\frac{23}{8} + \frac{24}{8}\right)\left(\frac{23}{8} - \frac{24}{8}\right) \\3 &= a\left(\frac{47}{8}\right)\left(-\frac{1}{8}\right) \\3 &= a\left(-\frac{47}{64}\right) \\a &= 3\left(-\frac{64}{47}\right) \\a &= -\frac{192}{47}\end{aligned}$$

Therefore, the quadratic function that models the shape of the fountain is given by

$$h = -\frac{192}{47}(x + 3)(x - 3).$$

## KEY IDEA

When determining a quadratic function that models a real-life situation, always begin by drawing a diagram. This step is essential in making sure you understand the information you have been given.

Pay attention to the variables you are expected to have in your function and their units (you may need to convert units to get a meaningful result).

## PRACTICE

(Answers below.)

6. Greg is part of a team of divers in South America looking for pearls. When they are submerged under water, they only have 10 minutes before their air supply runs out completely. After 30 seconds, Greg can reach 19 feet beneath the surface. Determine a quadratic function that models Greg's position  $p$  relative to the surface of the water, in feet, of his deepest dive after  $t$  minutes.
7. The profit of a theme park increases as ticket prices go up. When tickets were only \$10, the theme park made an average of \$3900 each day. When the theme park director made the tickets any more than \$20, he realized that he began to make less money because so many customers began refusing to pay.
  - (a) Determine a quadratic function that models the average profit per day  $p$  of the company in dollars in relation to the price  $t$  of the tickets in dollars.
  - (b) What is the maximum average profit per day of the company?

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Answers: 6.  $p = 4t(t - 10)$  7.(a)  $p = -13(t - 20)^2 + 5200$  (b) \$5200

## RECAP OF FOCUS QUESTION

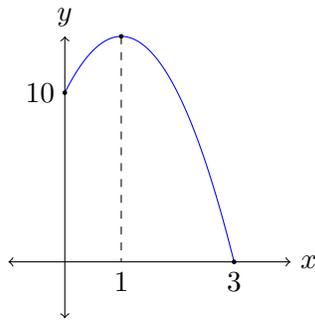
Recall the focus question, which was asked earlier in the lesson.

**Krissa is on the competitive diving team at her school. When jumping from a 10 metre diving board, she reaches her highest point after 1 second and hits the surface of the water after 3 seconds have passed. Determine a formula for the quadratic function that models Krissa's height  $h$  in metres above the surface of the water after  $t$  seconds?**

**In order to perform a new diving trick, Krissa's instructor tells her that she needs at least 13 metres of space above the surface of the water. Using the information given, will Krissa be able to perform the new trick?**

## SOLUTION

This situation is shown in the following diagram:



Notice that we are given two points on the parabola and we can quickly deduce that the equation of the axis of symmetry is  $x = 1$ . This means that we are lacking the values of  $a$  and  $q$  if we consider the vertex form of a quadratic equation,  $h = a(t - p)^2 + q = a(t - 1)^2 + q$ .

We use the values we have been given to produce two equations for the two unknown values  $a$  and  $q$ :

When  $t = 0$ ,  $h = 10$ , so

$$\begin{aligned} h &= a(t - 1)^2 + q \\ 10 &= a[0 - 1]^2 + q \\ 10 &= a + q \end{aligned}$$

When  $t = 3$ ,  $h = 0$ , so

$$\begin{aligned} h &= a(t - 1)^2 + q \\ 0 &= a(3 - 1)^2 + q \\ 0 &= 4a + q \end{aligned}$$

Subtracting the first equation from the second equation allows us to eliminate  $q$  and solve for  $a$ :

$$\begin{aligned} 0 - 10 &= (4a + q) - (a + q) \\ -10 &= 3a \\ a &= -\frac{10}{3} \end{aligned}$$

Substituting the value of  $a$  into either of the equations (let's use the first equation) allows us to solve for  $q$ :

$$\begin{aligned} 10 &= a + q \\ 10 &= -\frac{10}{3} + q \\ q &= 10 + \frac{10}{3} \\ q &= \frac{30}{3} + \frac{10}{3} \\ q &= \frac{40}{3} \end{aligned}$$

Therefore, the function that models Krissa's dive is  $h = -\frac{10}{3}(t - 1)^2 + \frac{40}{3}$ .

Now consider the second part to the focus question. Since Krissa needs at least 13 metres of space above the surface of the water, the maximum height she reaches in her dive must be higher than 13 metres. Therefore, to answer this question, we need to find Krissa's maximum height above the water.

Looking at the function we just determined, observe that the vertex of the parabola occurs at  $(1, 40/3)$ . Therefore Krissa's maximum height above the water is  $40/3$  metres, which is approximately 13.33 metres. Therefore, Krissa will be able to do the new trick.

Our focus question reminds us that most times we will want to determine the formula for a quadratic function in order to determine other important information about a real-life situation. The following problems will challenge you to put what we've learned throughout this module together with the lessons you have previously encountered.

### EXAMPLE 8

Recall the question in a previous example. The reason Nicole wanted to determine the formula for the quadratic function that modelled the shape of the fountain was to find its height. Calculate the maximum height of the water above ground if the fountain is sitting in a pond that is 2 feet high.

### SOLUTION

The function that models the shape of the fountain is  $h = -\frac{192}{47}(x - 3)(x + 3)$ . If we want to be able to find the maximum height, we will need to rearrange this equation into vertex form:

$$h = -\frac{192}{47}(x - 3)(x + 3)$$

$$h = -\frac{192}{47}(x^2 - 9)$$

$$h = -\frac{192}{47}x^2 + \frac{1728}{47}$$

Therefore, the maximum height of the water above the pond is  $\frac{1728}{47} \approx 37$  feet. Since the pond is 2 feet above ground, the top of the fountain reaches approximately 39 feet above the ground.

Like Example 8, the following exercises ask you to not only determine quadratic functions, but to also apply your understanding of parabolas to answer other questions about real-life situations.

## PRACTICE

(Answers below.)

8. Sara is the goalie of her soccer team. When she drop-kicks the soccer ball, it leaves her foot 80 cm from the ground and reaches a maximum height of 2510 cm after 4.5 seconds.
- Determine a formula for the quadratic function that models the height  $h$  in metres of the soccer ball  $t$  seconds after it has left Sara's foot.
  - When does the ball hit the ground?
9. A company wants to have pens produced to promote itself to customers. The cost of each pen decreases as more pens are produced, but eventually this price goes back up due to the extra labour costs the production plant encounters. For example, if the company purchases 1000 pens, the cost will be \$1.60 per pen. If the company purchases 3000 pens, the cost is \$1.00 per pen. The company will pay the cheapest price per unit only if they order 5000 pens.
- Determine a quadratic function that models the cost per pen  $C$  in dollars for the number of pens  $n$  produced.
  - The company decides to order 500 pens to start. It is not long before the pens become very popular with customers! A couple of months later, the company orders 1000 more. How much money would be saved if 1500 pens were ordered the first time?
10. An archway is going to be built above a single lane roadway in order to support a bridge. The arch will be 4 m in width and 6 m in height.
- Determine a quadratic function that models the height  $h$  of the archway in metres,  $n$  metres away from the centre.
  - A truck needs to pass under this bridge. What is the maximum height it can be if its width is 2.4 m?

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Answers: 8.(a)  $h = -1.2(t - 4.5)^2 + 25.1$  (b) about 9.1 s 9.(a)  $C = \frac{1}{2000000}(n - 5000)^2 + 0.8$  (b) \$387.50  
10.(a)  $h = -1.5n^2 + 6$  (b) about 3.8 m

## WWW

- What we did: We learned how to determine a formula for a quadratic function given various information.
- Why we did it: Quadratic functions are used to model numerous real-life situations. Determining such functions is often helpful in understanding and interpreting specific scenarios.
- What's next: Learning how to approach word problems that involve quadratic functions; learning how to use calculus to analyze quadratic functions.

## EXERCISES

11. Determine a formula for the quadratic function given the coordinates of its vertex  $(p, q)$  and a point it passes through,  $(x, y)$ .
  - (a)  $(p, q) = (6, 15)$ ,  $(x, y) = (0, 17)$
  - (b)  $(p, q) = (-8, -11)$ ,  $(x, y) = (-6, -27)$
  - (c)  $(p, q) = (1/2, -7/4)$ ,  $(x, y) = (1, -2)$
12. Determine a formula for a quadratic function given its zeros and a point its graph passes through.
  - (a)  $x = -5$  and  $x = 4/3$ ;  $A(3, 120)$
  - (b)  $x = -4$  and  $x = 1$ ;  $B(-12, -13)$
  - (c)  $x = -1/2$  and  $x = 9$ ;  $C(3, -42)$
13. Determine a formula for a quadratic function given two points on its graph and the equation of its axis of symmetry.
  - (a)  $D(2, 16)$ ,  $E(-1, 46)$ ;  $x = 3$
  - (b)  $F(0, -5)$ ,  $G(1, -8)$ ;  $x = -1$
  - (c)  $H(9, 17)$ ,  $I(18, 53)$ ;  $x = 12$
14. A flare is shot from a height of 287.5 cm above the surface of a lake. It appears to start falling back down about 6 m away from the boat and lands in the water only 11.5 m away from the boat.
  - (a) Determine a quadratic function that models the height  $h$  in metres of the flare above the water in relation to its distance  $d$  metres from the boat.
  - (b) What is the maximum height of the flare?

### CHALLENGE PROBLEM

15. After school, Linda and Xavier go to the local park and take turns seeing how far they can jump off of the swings. When Linda leaves the swing, she is 5 feet above the ground. She reaches her maximum height of 6 feet only 2 feet away from the swing. When Xavier leaves the swing, he is 6.5 feet above the ground and reaches his maximum height of 6.75 feet only 1 ft away from the swing.
- (a) Determine a quadratic function that models Linda's height  $h$  in feet above the ground versus her distance  $d$  in feet from the swing.
  - (b) Construct a second quadratic function that models Xavier's height  $h$  in feet above the ground versus his distance  $d$  in feet from the swing.
  - (c) Who travels the farthest distance away from the swing set?
16. Sea Otters dive for their food in the North Pacific Ocean. On average, they are able to hold their breath for 5 minutes under water. If an otter can reach 10 feet beneath the surface in only 6 seconds:
- (a) Determine a quadratic function that models its position  $p$  in feet below the surface after time  $t$  in seconds of its deepest dive.
  - (b) What is the maximum depth below the surface this otter can reach?