

# BROCK UNIVERSITY MATHEMATICS MODULES

## 11A2.2 Introduction to Factoring a Polynomial

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### WWW

- What it is: Expressing a polynomial as a product of polynomials of smaller degree.
- Why you need it: Factoring a polynomial aids in analyzing its behaviour, including analyzing its graph. Factoring a polynomial also helps in solving equations involving polynomial expressions.
- When to use it: Use factoring as a first attempt when analyzing polynomial functions and solving equations involving polynomials.

### PREREQUISITES

Before you tackle this module, make sure you have completed these modules:

**Expanding Polynomials \*\*\***, **Common Factoring \*\*\***

### WARMUP

Before you tackle this module, make sure you can solve the following exercises. If you have difficulties, please review the appropriate prerequisite modules.

(Answers below.)

1. Factor each expression.

- (a)  $8m^3 - 4m^2$    (b)  $20a^4 - 40a^5$    (c)  $5x^2y^2 + 15x^3y^2 - 10x^3y^2$   
(d)  $2a^4b^2c^5 - 10a^3b^3c^5d$    (e)  $7a^2b^4 + 14ab^5 + 21a^3b^3$    (f)  $12s^3t^6u^4v^2 - 36s^4t^5u^5$   
(g)  $3xy - 12xy^2z + 21x^2yz^2$    (h)  $m^3n^2p^5q^4 - 3m^2p^3q^5 + 4p^3q^3$

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Answers: 1. (a)  $4m^2(2m-1)$    (b)  $20a^4(1-2a)$    (c)  $5x^2y^2(1+x)$    (d)  $2a^3b^2c^5(a-5bd)$    (e)  $7ab^3(ab+2b^2+3a^2)$   
(f)  $12s^3t^5u^4(tv^2 - 3su)$    (g)  $3xy(1 - 4yz + 7xz^2)$    (h)  $p^3q^3(m^3n^2p^2q - 3m^2q^2 + 4)$

### Introduction

In this module, you will learn how to factor a variety of polynomial expressions, with a primary focus on trinomials. Factoring is used as a means of simplifying an expression into a product of its simplest parts. Factored form assists in the analysis of the behaviour of a polynomial function, including analyzing its graph. Factoring is also helpful when determining the coördinates of specific points on a polynomial function's graph (for example, maximum or minimum points), as well as solving equations involving polynomial expressions.

You will learn in subsequent modules that factoring applies particularly to finding important  $x$ -values of a function known as zeros, which are a key component in graphing and function analysis.

Before we start let's consider a geometric interpretation to present an alternative, visual representation of the factoring algorithm.

Expanding a product of binomials can be expressed geometrically using an "area model" of multiplication. So, consider a kitchen floor with an array of tiles. Each floor tile is square and measures 1 unit by 1 unit and as such has an area of 1 square unit.

If you have an area of tiles that measures 7 units long by 5 units wide, then the number of tiles is 35, which is also the area of the tiled part of the floor.

Similarly, if you have a floor measuring  $(4 + 3)$  tiles wide by  $(4 + 1)$  tiles long, the area would once again be 35.

Now, if we draw a horizontal line and a vertical line, we can break up the same floor into four rectangular pieces, the first measuring 4 by 4, the second 4 by 1, the third 3 by 4 and the fourth 3 by 1.

Now let's consider a similar diagram, but this time rather than measuring  $(4+3)$  by  $(4+1)$  let's assign the dimensions  $(x + 3)$  and  $(x + 1)$  to the width and length respectively. We can create the same four rectangular pieces, measuring  $x^2$ ,  $x(1)$ ,  $(3)x$ , and  $(3)(1)$  respectively. Therefore, we can see that when we expand a factored expression we get four terms and as such  $(x + 3)(x + 1)$  does **not** equal  $x^2 + (3)(1)$  but rather  $x^2 + x(1) + (3)x + (3)(1)$ , which when simplified equals  $x^2 + 4x + 3$ .

The algebraic analogue of this geometric observation is that when you expand  $(x + 3)(x + 1)$ , the coefficient of  $x$  is  $3 + 1$ , and the constant coefficient is  $(3)(1)$ . This observation can be generalized and expressed from a factoring perspective as follows:

### KEY IDEA

A quadratic expression of the form  $x^2 + bx + c$  can be factored in the form  $(x + s)(x + t)$  if the following two conditions are satisfied:

1.  $s + t = b$
2.  $st = c$

Thus, if you wish to factor an expression of the form  $x^2 + bx + c$ , seek two numbers that add to give  $b$  and that multiply to give  $c$ . Keeping this key idea in mind, let's look at the following example.

### EXAMPLE 1

Factor the trinomial  $x^2 + 11x + 18$ .

### SOLUTION

Given the trinomial  $x^2 + 11x + 18$  (i.e.  $b = 11$  and  $c = 18$ ) we must find two numbers that multiply to give 18 and add to give 11. Let's hope the two numbers are both integers, which would make our task easier. (If they are not integers, we might have to resort to the quadratic formula, which is discussed in Module 11A2.8.) Because  $18 > 0$ , either the two numbers are both positive or both negative. However, the two numbers must add to give 11, so they must both be positive.

Steps:

1. First consider those positive integers that multiply to give 18 (i.e.,  $c$ ):  $1 \times 18$ ,  $2 \times 9$ , and  $3 \times 6$ .
2. Now, determine whether the sum of any of these pairs of numbers equals 11 (i.e.,  $b$ ).  
 $1 + 18 = 19$   
 $2 + 9 = 11$   
 $3 + 6 = 9$
3. Checking the list in the previous step, we notice that there is a pair of whole numbers, namely 2 and 9, that satisfy both conditions:  $2 + 9 = 11$ , and  $2 \times 9 = 18$ . Therefore,  $x^2 + 11x + 18 = (x + 2)(x + 9)$ .
4. Finally you can check your work by expanding  $(x + 2)(x + 9)$  and verifying that it really does equal  $x^2 + 11x + 18$ .

Let's consider a few more examples.

### EXAMPLE 2

Factor the trinomial  $x^2 + 8x + 12$ .

### SOLUTION

We need to find two numbers that when multiplied give 12 and when added give 8. So we look at the pairs 1 & 12, 2 & 6, and 3 & 4, which are the positive factors of 12. (We don't bother with the negative factors of 12, because we know that the factors must either be both positive or both negative; however we can rule out the negative possibilities, because the sum of the factors is 8, which is positive.)

Notice that  $1 + 12 \neq 8$ ,  $3 + 4 \neq 8$ , but  $2 + 6 = 8$ . This means that the numbers that satisfy both conditions are 2 and 6, because  $2 + 6 = 8$  and  $2 \times 6 = 12$ .

Therefore, the solution is  $x^2 + 8x + 12 = (x + 2)(x + 6)$ .

As usual, you can check your work by expanding the right side of the previous equation and verifying that it really does equal the left side.

### EXAMPLE 3

Factor the trinomial  $y^2 - 3y - 28$ .

### SOLUTION

We need to find two numbers that multiply to give  $-28$  and add to give  $-3$ . This means that one of the numbers must be positive and the other must be negative. The integers factors of  $-28$  are  $-28$  and  $1$ ,  $-14$  and  $2$ ,  $-7$  and  $4$ ,  $-4$  and  $7$ ,  $-2$  and  $14$ , and finally,  $-1$  and  $28$ . By adding each pair of numbers together, we find that there is one pair that adds to  $-3$ , namely  $-7$  and  $4$ . The solution is thus  $y^2 - 3y - 28 = (y + 4)(y - 7)$ . Check by expanding the right side!

### EXAMPLE 4

Factor the trinomial  $x^2 - 12x + 20$ .

### SOLUTION

Given the trinomial  $x^2 - 12x + 20$  we require two numbers with a product of  $20$  and a sum of  $-12$ . We find that the two numbers that satisfy both conditions are  $-2$  and  $-10$ .

Thus, the solution is  $x^2 - 12x + 20 = (x - 2)(x - 10)$ . Check by expanding the right side!

The previous examples allow us to note the following key ideas.

### KEY IDEA

If  $x^2 + bx + c$  can be factored as  $(x + s)(x + t)$ , then:

- When  $c$  is positive, then the product of  $s$  and  $t$  is positive. This means that  $s$  and  $t$  are either both positive or both negative.
- When  $c$  is negative, then the product of  $s$  and  $t$  is negative. This means that  $s$  and  $t$  have opposite signs. Therefore if  $s$  is positive,  $t$  is negative, and vice versa.

Now it's your turn! Try these problems out on your own.

### PRACTICE

(Answers below.)

1. Factor each trinomial.

(a)  $a^2 - 13a + 42$    (b)  $t^2 + 8t + 16$    (c)  $a^2 + 14a + 24$   
(d)  $d^2 - 7d + 12$    (e)  $y^2 - 2y - 8$    (f)  $k^2 - k - 6$

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Answers: 1.(a)  $(a - 7)(a - 6)$    (b)  $(t + 4)(t + 4) = (t + 4)^2$    (c)  $(a + 12)(a + 2)$    (d)  $(d - 4)(d - 3)$   
(e)  $(y + 2)(y - 4)$    (f)  $(k + 2)(k - 3)$

Trinomials are not always quadratic in nature, having powers greater than 2, and sometimes they involve more than one variable. However, in certain cases we can use the same steps as learned above to factor these “special” trinomials. We’ll strive to follow the same steps and process as outlined above to the subsequent examples.

It may help you to recall that when terms with the same base are multiplied, the base remains the same and the exponents are added.

Let’s consider a few examples to illustrate this concept.

### EXAMPLE 5

Factor the trinomial  $x^2 + 7xy + 12y^2$ .

#### SOLUTION

Consider the trinomial  $x^2 + 7x + 12$ . Using the methods discussed earlier in this module, we can factor this trinomial to obtain  $x^2 + 7x + 12 = (x + 3)(x + 4)$ . This trinomial is similar to the one we are asked to factor, which makes us wonder whether we could just tinker with the factors we just obtained to guess factors for the trinomial  $x^2 + 7xy + 12y^2$ .

**First guess:**  $x^2 + 7xy + 12y^2 = (x + 3y^2)(x + 4)$ . Maybe by including a  $y^2$  in the first factor this will work. Expanding the right side shows that this does **not** work:  $(x + 3y^2)(x + 4) = x^2 + 4x + 3xy^2 + 12y^2$ . The first and fourth terms are correct, but the middle two terms are incorrect.

**Second guess:**  $x^2 + 7xy + 12y^2 = (x + 3)(x + 4y^2)$ . This does **not** work for a similar reason as the first guess:  $(x + 3)(x + 4y^2) = x^2 + 4xy^2 + 3x + 12y^2$ . The first and fourth terms are correct, but the middle two terms are incorrect.

**Third guess:** We need a  $y^2$  term; another way to achieve this is to include a factor of  $y$  in each of the parentheses in the factored form. So our third guess is:  $x^2 + 7xy + 12y^2 = (x + 3y)(x + 4y)$ . This **does** work, as you can see by expanding the parentheses:  $(x + 3y)(x + 4y) = x^2 + 4xy + 3xy + 12y^2 = x^2 + 7xy + 12y^2$ .

What we’ve learned from this example is that it is possible to factor trinomials in two variables using the same method as factoring a trinomial in one variable, as long as the trinomial has the right form.

### EXAMPLE 6

Factor the trinomial  $w^2 + 14uw + 40u^2$ .

#### SOLUTION

The form of the trinomial is the same as in the previous example. So let’s proceed using the same method, searching for two numbers that add to 14 and multiply to give 40. After some thought, you will see that the two needed numbers are 4 and 10. Thus,

$$w^2 + 14uw + 40u^2 = (w + 4u)(w + 10u)$$

Check the result by expanding the right side.

### EXAMPLE 7

Factor the trinomial  $x^2y^2 - 10xy + 24$ .

### SOLUTION

This is somewhat similar in form to the previous examples, but it is a little different. If we search for two numbers that add to give  $-10$  and multiply to give  $24$ , we will be successful:  $-6$  and  $-4$ . This gives us the confidence to try a few guesses:

**First guess:** One way to produce the  $x^2y^2$  term is to include an  $x^2$  term in the first parenthesis and a  $y^2$  term in the second parenthesis, like this:  $x^2y^2 - 10xy + 24 = (x^2 - 6)(y^2 - 4)$ . Checking the guess by expanding, we see that it does **not** work:  $(x^2 - 6)(y^2 - 4) = x^2y^2 - 4x^2 - 6y^2 + 24$ . The first and fourth terms are correct, but the middle two terms are not correct.

**Second guess:** Another way to produce the  $x^2y^2$  term is to include an  $xy$  term in each parenthesis, like this:  $x^2y^2 - 10xy + 24 = (xy - 6)(xy - 4)$ . Checking the guess by expanding, we see that it **does** work:  $(xy - 6)(xy - 4) = x^2y^2 - 4xy - 6xy + 24 = x^2y^2 - 10xy + 24$ .

Let's now look at a trinomial for which the degree is greater than 2.

### EXAMPLE 8

Factor the trinomial  $x^4 - 8x^2y^2 - 20y^4$ .

### SOLUTION

If we attempt to use a method similar to the one used in the previous examples, we seek two numbers that add to give  $-8$  and multiply to give  $-20$ . After some searching, we find that the two numbers are  $-10$  and  $2$ .

What expressions should go inside the two parentheses? As a first guess, we could try  $x^2$  in each of the two parentheses, because their product is  $x^4$ , and then perhaps  $-10y^2$  in one parenthesis and  $2y^2$  in the other, because their product is  $-20y^4$ . Putting this all together, our guess is

$$x^4 - 8x^2y^2 - 20y^4 = (x^2 - 10y^2)(x^2 + 2y^2)$$

Expanding the right side, we see that the guess works! (Check it for yourself!)

Now let's see if you can apply everything you've learned so far to the following exercises.

### PRACTICE

(Answers below.)

2. Factor each trinomial.

(a)  $m^2 + 9mn + 18n^2$    (b)  $s^2 + 20st + 36t^2$    (c)  $c^2d^2 + 21cd - 100$   
(d)  $x^4 - 29x^2y^2 + 100y^4$    (e)  $y^4 - 5y^2 - 36$    (f)  $p^2q^2 + 18pq + 80$

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Answers: (a)  $(m + 6n)(m + 3n)$    (b)  $(s + 18t)(s + 2t)$    (c)  $(cd + 25)(cd - 4)$   
(d)  $(x^2 - 25y^2)(x^2 - 4y^2)$    (e)  $(y^2 - 9)(y^2 + 4)$    (f)  $(pq + 8)(pq + 10)$

So far we've only looked at trinomials where the  $x^2$  term has a coefficient of 1. However, generally trinomials are of the form  $y = ax^2 + bx + c$  where  $a$ , the coefficient of the  $x^2$  term, is not necessarily equal to 1. This leads to the following observation.

### KEY IDEA

A quadratic, trinomial expression of the form  $y = ax^2 + bx + c$  can be factored in the form  $y = (qx + s)(rx + t)$ , if the following three conditions are satisfied:

1.  $qr = a$
2.  $st = c$
3.  $qt + rs = b$

Note that when compared to the conditions that must be satisfied for factoring simple trinomials, the second condition is the same as before, and the new condition 1 is similar, but the third condition is a little more complicated. This complication makes such trinomials “tricky” and hence more difficult to factor if we rely on pattern recognition.

We will present two methods for factoring tricky trinomials. Select and use whichever one is easiest for you to understand and works best for you.

Method 1: “Trial and Error Method”

At first this will require you to simply guess and check, however with time you'll find that guessing becomes less of a factor and it will become more intuitive. Also, despite the perceived abstractness of the method, you'll find that the guess is not as random as one would think, but rather requires pattern recognition and deduction.

Steps:

1. Check for a common factor first. You may find that after common factoring, the trinomial is not actually tricky but rather simple in nature.

2. Choose two factors that multiply to produce the correct first coefficient (i.e.  $a$ ). These numbers will take the place of  $q$  and  $r$  in the factored form outlined in the key-idea box.

For example, consider  $5z^2 + 14z - 3$ . One could guess that two factors that multiply to give the coefficient of  $a$  are  $(5)(1) = 5$ . (Of course, one could also guess that the factors are  $(-5)(-1) = 5$ , but let's try the first guess first.)

3. Choose two factors that multiply to produce the correct constant coefficient (i.e.  $c$ ). These numbers will take the place of  $s$  and  $t$  in the factored form outlined in the key-idea box. For instance, we could choose  $(3)(-1) = -3$ .

4. Check that the factors that you have selected can be arranged so that they give the correct coefficient of the linear term (i.e.  $b$ ). In our example, let's try  $(5)(-1) + (1)(3) = -5 + 3 = -2 \neq b$ , which does **not** work. A second try is  $(5)(3) + (1)(-1) = 15 - 1 = 14 = b$ , which **does** work.

Therefore, the solution is  $5z^2 + 14z - 3 = (5z - 1)(z + 3)$ . Check that this is valid by expanding the right side.

Method 2: “Common Factor Method”

This is a systematic procedure that does not require any guessing.

Steps:

1. Multiply the first and third coefficients together (i.e.  $a$  and  $c$ ) — let's call the resulting number  $d$ . For example, once again consider  $5z^2 + 14z - 3$ . Multiplying the first and third coefficients together results in  $d = 5(-3) = -15$ .

2. Determine two factors of  $d$  that when added give  $b$ . After some thought, you will come up with 15 and  $-1$ , because  $b = 14 = -1 + 15$ .

3. Now write the expression again, this time replacing the linear term by two terms corresponding to the numbers obtained in Step 2. For our example, this means  $5z^2 + 14z - 3 = 5z^2 + 15z - z - 3$ .

4. Group the first two terms together, and group the third and fourth terms together. Common factor, if possible, within each group. Note that the remaining terms will have a common factor. In our example,  $5z^2 + 15z - z - 3 = 5z(z + 3) - (z + 3)$ .

5. Common factor once more to obtain the final result. For our example,  $5z(z + 3) - (z + 3) = (5z - 1)(z + 3)$ . Thus, the final result is  $5z^2 + 14z - 3 = (5z - 1)(z + 3)$ .

Notice that the result of each method is the same, as you can check by expanding. As the thought process behind Method 1 is unique to every individual, and is thus difficult to document, let's consider a few more examples using Method 2.

### EXAMPLE 9

Factor  $3x^2 - 10x + 8$ .

#### SOLUTION

Notice that  $a = 3$ ,  $b = -10$ , and  $c = 8$ . Multiplying  $a$  and  $c$  together, we observe that  $d = (3)(8) = 24$ .

Now if we look for two factors of  $d = 24$  such that their sum gives  $b = -10$ , we discover that the necessary numbers are  $-6$  and  $-4$ , because  $(-6)(-4) = 24 = d$  and  $-6 + (-4) = -10 = b$ .

Rewriting our expression, replacing the linear term by two terms that correspond to the numbers  $-6$  and  $-4$ , we get  $3x^2 - 6x - 4x + 8$ .

Grouping and common factoring, this results in  $3x(x - 2) - 4(x - 2)$ .

Common factoring once more, we find the solution to be  $3x^2 - 10x + 8 = (x - 2)(3x - 4)$ . Check the final result by expanding.

### EXAMPLE 10

Factor  $2y^2 - 5y - 12$ .

#### SOLUTION

Note that  $a = 2$ ,  $b = -5$  and  $c = -12$ . Then  $d = ac = (2)(-12) = -24$ .

Considering divisors of  $-24$ , we can deduce that 3 and  $-8$  are the required numbers, because  $(3)(-8) = -24$  and  $3 + (-8) = -5$ .

Rewriting our expression, we get  $2y^2 - 5y - 12 = 2y^2 + 3y - 8y - 12$ .

Grouping and common factoring results in  $2y^2 + 3y - 8y - 12 = y(2y + 3) - 4(2y + 3)$ .

Common factoring once more we get the solution  $2y^2 - 5y - 12 = (2y + 3)(y - 4)$ . Check the final result by expanding.

### EXAMPLE 11

Factor  $9x^2 + 12x + 4$ .

### SOLUTION

Note that  $a = 9$ ,  $b = 12$ , and  $c = 4$ . Then  $d = ac = (9)(4) = 36$ .

From the value of  $d$  we can deduce that the factors that satisfy the required conditions are 6 and 6, because  $(6)(6) = 36 = d$  and  $6 + 6 = 12 = b$ .

The rewritten expression yields  $9x^2 + 6x + 6x + 4$  and after grouping and common factoring results in  $3x(3x + 2) + 2(3x + 2)$ .

After common factoring one last time, we find the solution to be  $9x^2 + 12x + 4 = (3x + 2)(3x + 2) = (3x + 2)^2$ . Check by expanding!

Now it's time for you to practice! Remember to use whichever method you prefer and makes the most sense to you.

### PRACTICE

(Answers below.)

3. Factor each trinomial. (a)  $2x^2 + 5x + 3$  (b)  $6a^2 + 7a + 2$  (c)  $21c^2 + 20c + 4$   
(d)  $10s^2 + 23s - 5$  (e)  $3q^2 - 5q - 2$  (f)  $6p^2 - p - 12$

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Answers: (a)  $(2x+3)(x+1)$  (b)  $(3x+2)(2x+1)$  (c)  $(7x+2)(3x+2)$  (d)  $(5x-1)(2x+5)$  (e)  $(3q+1)(q-2)$   
(f)  $(3p+4)(2p-3)$

### KEY IDEA

Though we didn't see any examples of this above, as mentioned with Step 1 of the first method, it's important to always look for common factors first. This may make what appears to be a tricky trinomial into a simple trinomial, or reduce the size of the coefficients and thus simplify the task of factoring the expression. To illustrate this point try the following exercises.

### PRACTICE

(Answers below.)

4. Factor each trinomial. (a)  $3x^2 - 12xy - 96y^2$  (b)  $4x^2 - 28x + 48$  (c)  $2x^2y + 4xy - 48y$   
(d)  $4a^2b^2 - 12ab - 40$

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Answers: (a)  $3(x-8y)(x+4y)$  (b)  $4(x-4)(x-3)$  (c)  $2y(x-4)(x+6)$  (d)  $4(ab-5)(ab+2)$

We will now examine one last factoring pattern known as the “difference of squares” factoring pattern. In the title we see the word “square,” a geometric shape that we know to have sides of

equal length. The algebraic analogue of this is a number that is the product of two equal numbers. Note this concept is not limited merely to numbers, as it includes quantities as well. For example,  $x^2$  can be written as  $(x)(x)$ , and as such is also a square.

Recall that the whole “square numbers” are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, . . .

Consider quadratic expressions of the form  $x^2 - k^2$ .

You may have noticed already that this differs slightly from the general form  $y = x^2 + bx + c$ , as in this case  $b = 0$  and as such there is no linear term. We’re left simply with a binomial expression. We’re also writing  $c$  as  $-k^2$  to guide us to think about a difference of squares.

Thinking back to our kitchen floor tiles, for  $a$  to equal 0, our factors  $s$  and  $t$  must have the same absolute values, but with opposite signs so that when the factored expression is expanded the  $x$ -terms will cancel, leaving only the  $x^2$ -term and constant term.

To illustrate this, observe that when we expand the factored expression  $(x - 3)(x + 3)$ , we get  $(x)(x) + (x)(3) - (3)(x) - (3)(3)$ , and after collecting like terms, the simplified expression yields  $x^2 + 3x - 3x - 9 = x^2 - 9$ .

### KEY IDEA

A quadratic expression of the form  $y = x^2 - k^2$  can be factored in the form  $(x - k)(x + k)$ .

Consider these examples to help solidify the concept.

### EXAMPLE 12

Factor  $100 - y^2$ .

### SOLUTION

Looking at the expression  $100 - y^2$  note that  $100 = (10)(10) = (10)^2$ , so the expression is in the form of a difference of squares.

Therefore, we can deduce that  $100 - y^2 = (10 - y)(10 + y)$ . Check by expanding, and notice how the linear terms cancel so that there is no linear term in the final expanded expression.

### EXAMPLE 13

Factor  $y^2 - 81x^4$ .

### SOLUTION

We recognize the first term as a square, but the second term is a little more complicated. By recognizing that  $81 = 9^2$  and  $x^4 = (x^2)^2$ , we can write  $(81)(x^4)$  as  $(9)^2(x^2)^2$  or equivalently as  $((9)(x^2))^2$ . Therefore, using the difference-of-squares factoring pattern, we can write

$$y^2 - 81x^4 = (y - 9x^2)(y + 9x^2)$$

**EXAMPLE 14**Factor  $16x^2y^2 - 25b^2$ .**SOLUTION**

We can recognize the first term as a square by writing it as  $16x^2y^2 - 25b^2 = 4^2x^2y^2 - 5^2b^2 = (4xy)^2 - (5b)^2$ . Therefore, using the difference-of-squares factoring pattern, we can write

Therefore, the solution is

$$16x^2y^2 - 25b^2 = (4xy - 5b)(4xy + 5b)$$

**EXAMPLE 15**Factor  $(2x + 5)^2 - 81x^2$ .**SOLUTION**

This expression has the structure of a difference of squares; that is, it is in the form  $a^2 - b^2$ , where  $a^2 = (2x + 5)^2$ , and therefore  $a = (2x + 5)$ , and  $b^2 = 81x^2 = (9x)^2$ , and therefore  $b = 9x$ . Thus, according to the difference of squares pattern,

$$\begin{aligned} (2x + 5)^2 - 81x^2 &= (2x + 5)^2 - (9x)^2 \\ &= a^2 - b^2 \\ &= (a - b)(a + b) \\ &= (2x + 5 - 9x)(2x + 5 + 9x) \\ &= (-7x + 5)(11x + 5) \end{aligned}$$

Now you give it a try!

**PRACTICE**

(Answers below.)

5. Factor each expression. (a)  $x^2 - 49$  (b)  $121 - 36t^2$  (c)  $49c^2d^2 - \frac{81}{25}b^2$  (d)  $144x^4 - 1$   
 (e)  $(2x - y)^2 - (6x - 2y)^2$  (f)  $p^2q^2 - 81$  (g)  $4x - 16xy^2$  (h)  $27xy - 75a^2xy$

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Answers: (a)  $(x - 7)(x + 7)$  (b)  $(11 - 6t)(11 + 6t)$  (c)  $(7cd - \frac{9}{5}b)(7cd + \frac{9}{5}b)$  (d)  $(12x^2 - 1)(12x^2 + 1)$  (e)  $(-4x + y)(8x - 3y)$  (f)  $(pq - 9)(pq + 9)$  (g)  $4x(1 - 2y)(1 + 2y)$  (h)  $3xy(3 - 5a)(3 + 5a)$

## KEY IDEA

It's important to note that more than one factoring step may be needed to fully factor a polynomial expression. This may require a combination of the methods discussed throughout the lesson. Consider the following example: Suppose we wish to factor  $x^4 - 29x^2y^2 + 100y^4$ .

The first step is always to look for any common factors. As none exist, we proceed by treating this expression as a simple trinomial and factoring accordingly.

This gives  $x^4 - 29x^2y^2 + 100y^4 = (x^2 - 4y^2)(x^2 - 25y^2)$ .

Note that each of the factors is a difference of squares, and therefore each factor can be factored further. Using the difference of squares pattern for each factor, we get the final result  $x^4 - 29x^2y^2 + 100y^4 = (x + 2y)(x - 2y)(x + 5y)(x - 5y)$ .

## RECAP: Steps for factoring a trinomial

1. First check for common factors.
2. If there are two terms, check to see whether the expression fits the difference of squares pattern. If so, factor using this pattern.
3. If there are three terms, attempt to factor according to one of the trinomial patterns learned in this module.
4. Check to see if any of the newly found factors can be factored further.
5. Check your final result by expanding and comparing with the original expression.
6. Learn more factoring patterns, in case none of the ones used here works!
7. It's possible that your expression cannot be factored, or at least cannot be factored easily. Once again, learning some advanced mathematics will be helpful!

## WWW

- What we did: We learned how to factor simple, "special," and "tricky" trinomials as well as how to use the difference of squares method to factor specific binomials.
- Why we did it: Factoring polynomial expressions is helpful in solving certain equations, and in analyzing and plotting certain graphs.
- What's next: We can build on what we have learned here to solve polynomial equations, and to help us determine zeros of polynomial functions.

## EXERCISES

1. Factor each expression.

- (a)  $15y^3 - 35y^2$    (b)  $4 - 81x^2$    (c)  $x^2 + 5x - 66$   
(d)  $18a^2 - 50$    (e)  $5x^2 - 17x + 6$    (f)  $30x^3 + 35x^2 - 15x$